

## ESTIMATION OF EARTHQUAKE SOURCE PARAMETERS USING P AND S-WAVES SPECTRA – CASE STUDY THE 2012 SEISMIC CLUSTER IN SOFIA SEISMOGENIC ZONE

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DOI: 10.34975/bgj-2020.43.6

**Abstract:** Spectral analysis of seismic waves is one of the most important origin of information for the earthquake sources. In this study are presented P- and S-waves spectra of the  $M_w$ 5.6 Sofia 2012 earthquake seismic sequence. Earthquake source parameters (seismic moment, source radius, stress drop) are estimated applying Brune model. The model has been used extensively and it has been shown that it is in a good agreement with observations from many different tectonic regions and for a large range of magnitudes. The database is compiled using digital data from the Bulgarian Seismological Network - NOTSSI (National Operative Telemetric System for Seismological Information). Displacement spectra are generated for P- and S- waves to estimate some source parameters, such as seismic moment, stress drop, source radius and moment magnitude. The source parameters are estimated for 6 earthquakes: for the main event with moment magnitude  $M_w$ 5.6 and for five aftershocks with magnitude  $M_w$  in interval  $3.4 \div 4.5$ . The stress drop values estimated for P - waves are within the expected range for moderate earthquakes while for the S - waves the stress drop values are several times higher than the expected ones. The strongest aftershocks are characterized with lower than the main event stress drop values.

**Key words:** spectra, stress drop, source radius, seismic moment, Sofia seismogenic zone

### 1. Introduction

Seismology is the science that studies the seismic waves and what they tell us about the structure of the Earth and the physics of the earthquakes. It is the primary means by which scientists learn about Earth's deep interior, where direct observations

are impossible, and has provided many of the most important discoveries regarding the nature of our planet. It is also directly concerned with understanding the physical processes that cause earthquakes and seeking ways to reduce their destructive impacts on humanity.

The effects of earthquakes are function of a number of random factors ranging within broad limits, which should be adequately modelled on the basis of physical considerations, and the available seismic data. Modelling of space-time and energy distribution of earthquakes is a major aspect of modern seismological research. Examination of the space - time distribution of earthquakes is of fundamental importance for understanding the physics of the earthquake generation process. Another important aspect related to seismicity is spectral analysis of seismic waves. This analysis provides information of fundamental importance for parameters characterizing the earthquake source process. Earthquake parameters (seismic moment, source radius, stress drop) are estimated applying Brune model. The model has been used extensively and it has been shown that it gives a good agreement with observations from many different tectonic regions and for a large range of magnitudes.

In the present study are presented the spectra of P- and S-waves for earthquakes from the 2012 seismic cluster (main earthquake - aftershock).

A formal definition of seismic clusters is still lacking despite of the conception that the earthquake clustering is an essential aspect of seismicity that provides key information on earthquake dynamics (Zaliapin and Ben-Zion, 2013).

Aftershocks are defined as seismicity above the background activity following a main shock (Liu and Stein, 2011). Aftershocks occur after the main event and their frequency decays over time, typically following a pattern known as the Omori's law, which later is modified by Utsu (1961) and is known as modified Omori's law. The power-law decay represented by the modified Omori relation is an example of temporal self-similarity of the earthquake source process. The duration of aftershock sequences may last months, a few years, or even longer for earthquakes within stable continental interiors (Stein and Liu, 2009). Some authors recognize that the main causes of aftershocks include main shock-induced changes of frictional properties of the fault zone and stress perturbations (e.g. Liu and Stein, 2011).

In our study the source parameters of aftershock sequences of the May 22, 2012 earthquake with moment magnitude  $M_w 5.6$  ( $T_0=00:00:32$ ,  $\varphi=42.58$ ,  $\lambda=23.02$  and  $h=14$  km) are examined. The 2012  $M_w 5.6$  earthquake occurred in Sofia seismogenic zone after a long quiescence (of about 95 years) for moderate events. Moreover, a reduced number of small earthquakes have also been registered in the recent past. The 2012 earthquake is located in the vicinity of the city of Pernik at about 25 km south west of the city of Sofia. The quake was followed by intensive aftershock activity. A detailed analysis of the space - time distribution of aftershocks is presented in Solakov et. al. 2016. The 2012  $M_w 5.6$  earthquake was largely felt on the territory of Bulgaria and neighboring countries: northern Greece, northern Macedonia, eastern Serbia and southern Romania. No casualties and severe injuries have been reported. Predominantly moderate damages were observed in the epicentral area (in the cities of Pernik, Radomir and Sofia).

## 2. Method and Data

### 2.1 Method

Spectral analysis of seismic waves is one of the most important origin of information for the earthquake sources. The earthquake source parameters are computed following Brune's theory by using the corner frequency and the low frequency asymptote. The Brune model predicts the source displacement spectrum  $S(f)$ , which depends on  $M_0$  - the seismic moment,  $\rho$  - density,  $v$  - velocity at the source (P or S-velocity depending on spectrum), and  $f_0$  - corner frequency.

**The seismic moment  $M_0$**  (in N\*m) is a direct measure of the tectonic size. The scalar seismic moment  $M_0$  is defined by the equation:

$$M_0 = \mu SD, \quad (1)$$

where  $\mu$  is the shear modulus of the rocks involved in the earthquake (in pascals (Pa), i.e. newton per square meter),  $S$  - is the area of the rupture along the geologic fault where the earthquake occurred (in square meters), and  $D$  is the average slip (displacement offset between the two sides of the fault). The seismic moment can be determined by moment tensor inversion or spectral analysis. It is the most objective static measure of earthquake size and is used to determine moment magnitude.

**Stress drop  $\sigma$**  is the average difference between initial and final stress along a fault after an earthquake. For large, shallow earthquakes,  $\Delta\sigma$  vary from about 1 to 10 MPa or from 10 to 100 bars with  $M_0$  variations from  $10^{18}$  to  $10^{23}$  Nm (Kanamori and Anderson, 1975; Kanamori and Brodsky, 2004). It has been observed that earthquakes near plate boundaries (interplate events) generally have been observed to have somewhat lower stress drops than those that occur in the interior of plates (intraplate events) (e.g., Kanamori and Anderson, 1975; Kanamori and Allen, 1986). In average  $\Delta\sigma$  for interplate quakes is about 3 MPa (30 bars) while for intraplate events it is about 6 MPa (60 bars) (Allmann and Shearer, 2007).

For a circular fault in a whole space, Eshelby (1957) obtained:

$$\Delta\sigma = \frac{7}{16} \frac{M_0}{r^3}, \quad (2)$$

where  $r$  is the fault radius (in m or km) and  $M_0$  is seismic moment.

The first quantitative model for estimating stress drop was derived by Brune (1970), who assumed a simple kinematic model for a circular fault with effectively infinite rupture velocity and showed that the expected high-frequency spectral falloff rate is  $\omega^{-2}$  and that the corner frequency is inversely proportional to the source radius. This result, together with several other proposed rupture models, predicts that the fault radius varies as:

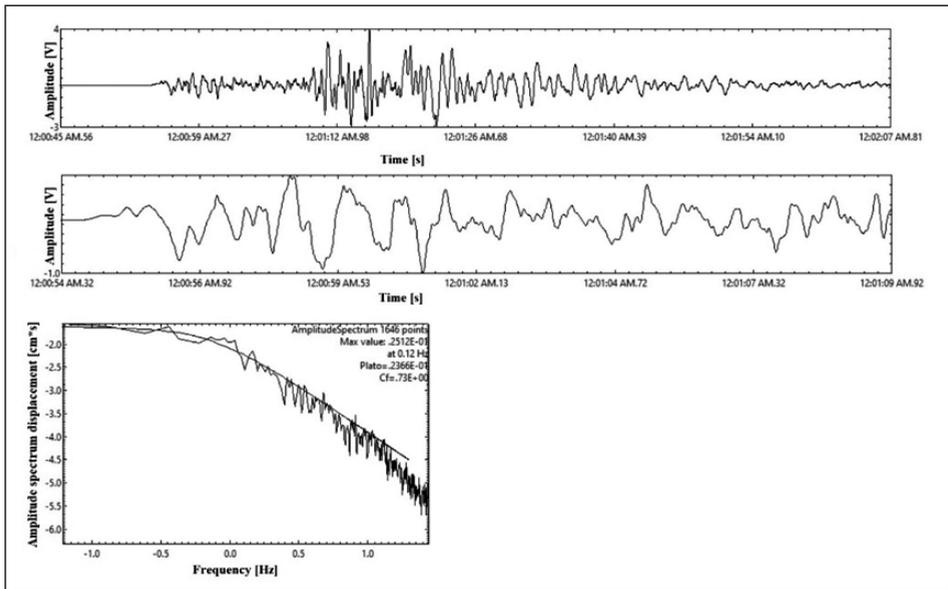
$$r = \frac{k\beta}{f_c}, \quad (3)$$

where  $r$  is the fault radius (in m or km),  $f_c$  is the observed corner frequency (in Hz) and  $k$  is a constant that depends upon the specific theoretical model.

To estimate the spectral parameters of the earthquake, it is necessary to transform the signal. The next figure presents an example of transformation of amplitude-time signal into amplitude-frequency and generated displacement spectra using Fast Fourier Transformation (FFT). In the figure 1 is presented: a) a time window of the P-wave signal from seismic station VTS; b) a time window of several seconds of the signal that are used for spectra generation; c) displacement spectra of P-wave.

The amplitude of seismic pulses in a perfectly elastic environment is controlled by the reflection and transmission of energy at the different boundaries. Seismic waves attenuate with time, the amplitudes of the waves are changed as they pass through different layers of the earth.

We study the distance effect on the low frequency spectral amplitude of the P and S waves using quality factor Q and near surface attenuation  $\kappa$ . In the study, we fixed  $Q = 400$  (Malagnini et al., 2000) and  $\kappa = 0.035$  (Margaris and Boore, 1998).



**Fig. 1.** Example of P-wave displacement spectra.

In the study the parameters: seismic moment  $M_0$ ; stress drop  $\Delta\sigma$ ; and source radius  $r$  are calculated using the following relationships.

The seismic moment for P-wave ( $M_{0p}$ ) is calculated as:

$$M_{0p} = \frac{\rho \cdot 4\pi \cdot \Omega_p \cdot R \cdot v_p^2}{R_{\theta\phi}(P)}, \quad (4)$$

where  $\rho$  – density in  $\text{g}/\text{cm}^3$ ,  $\Omega_p$  – spectral level in  $\text{nm} \cdot \text{s}$ ,  $v_p$  – velocity of P-wave in  $\text{km}/\text{s}$ ,  $R$  – distance in  $\text{km}$ ,  $R_{\theta\phi}(P)$  – radiation pattern.

The average correction for radiation pattern varies between 0.55 and 0.85 as presented in the literature. According to Aki and Richards, the average is 0.52 and 0.63 for P and S-waves, respectively. (Aki and Richards, 2002).

The seismic moment for S-wave ( $M_{0s}$ ) is calculated as:

$$M_{0s} = \frac{\rho \cdot 4\pi \cdot \Omega_s \cdot R \cdot v_s^2}{R_{\theta\phi}(S)}, \quad (5)$$

where  $\rho$  – density in  $\text{g/cm}^3$ ,  $\Omega_s$  – spectral level in  $\text{nm}^*\text{s}$ , which is determined by the spectra of wave, and its value is determined by Z, N, E – components ( $\Omega_s = \sqrt{\Omega_Z^2 + \Omega_N^2 + \Omega_E^2}$ ),  $R_{\theta\phi}(S)$  – radiation pattern, which for seismic moment for S-wave is 0.63, and  $v_s$  is velocity of S-wave in  $\text{km/s}$ .

The following formulas are used to determine the source radius and stress drop:

$$r_p = \frac{v_p \cdot 3.36}{2\pi f_0}, \quad (6)$$

where  $v_p$  is P-wave velocity, and  $f_0$  is corner frequency in Hz.

For S-wave, the source radius is determined in an identical approach:

$$r_s = \frac{v_s \cdot 2.34}{2\pi f_0}, \quad (7)$$

where  $v_s$  is S-wave velocity, and  $f_0$  is corner frequency in Hz.

The stress drops for P- and S-waves are calculated using the following relationships (Eshelby, 1957):

$$\Delta\sigma_p = \frac{7}{16} \frac{M_{0p}}{r_p^3}, \quad (8)$$

$$\Delta\sigma_s = \frac{7}{16} \frac{M_{0s}}{r_s^3}, \quad (9)$$

where  $r_p$  and  $r_s$  are the radius for P- and S-waves.

## 2.2 Input data

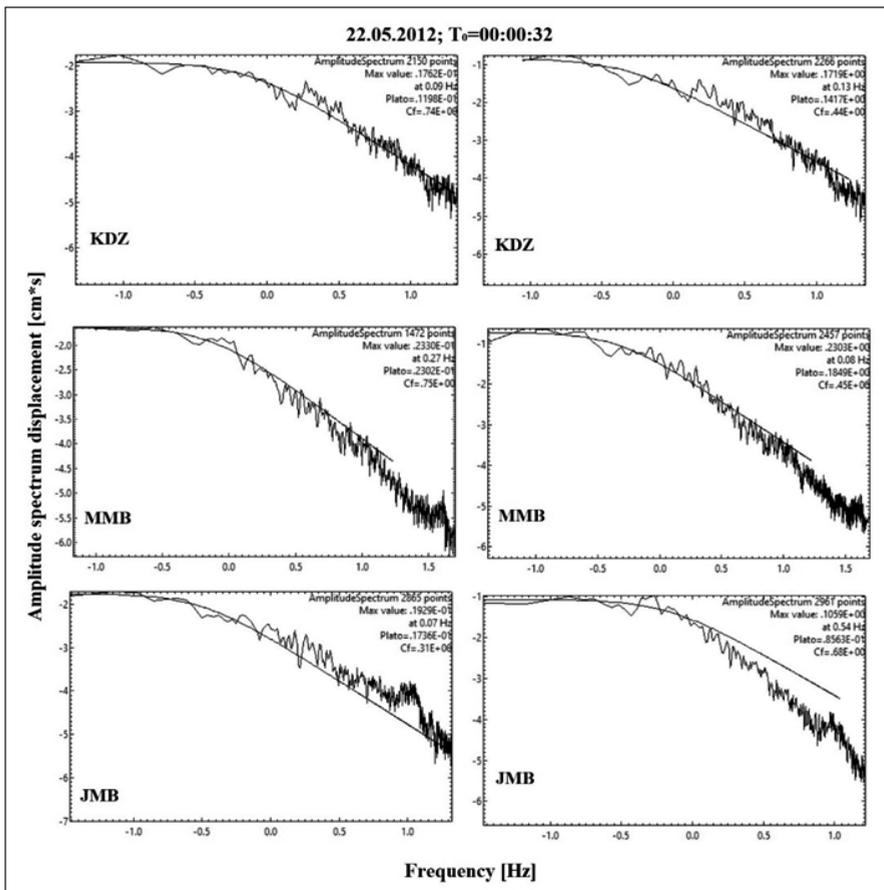
In the present study 218 digital records from 6 earthquakes (recorded at the stations of Bulgarian Seismological Network - NOTSSI) in the magnitude range  $M_w=3.4 - 5.6$  are analyzed. Spectra are generated on the base of records at the stations at a distance less than 200 km.

### 3. Results

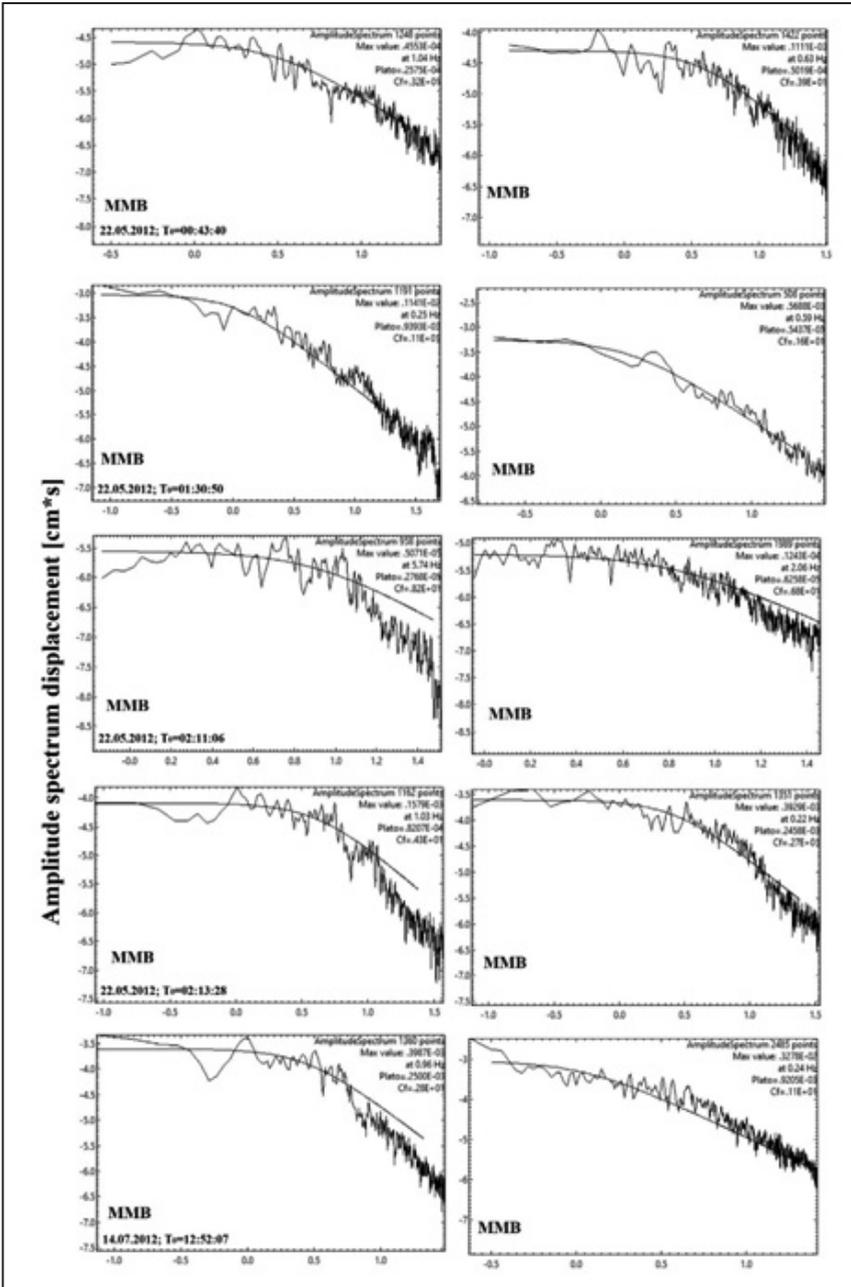
The results of the present study are presented in Fig. 2 - 3 and Tab. 1. Results of spectral analysis for the main event is presented in Fig. 2 and spectra of five of the strongest aftershocks are presented in Fig. 3.

The source parameters are estimated for 6 earthquakes: for the main event with magnitude  $M_w$  5.6 and for five aftershocks with magnitude  $M_w$  in the interval  $3.4 \div 4.5$ .

The spectra for the main event with  $M_w$  5.6 are generated on the base of the records at 13 stations. Displacement spectra for P and S waves based on records at 3 stations are presented in the Fig. 2. In the figure 3 are presented spectra for five of the strongest aftershocks. Presented displacement spectra for aftershocks both for P and S waves are based on records at the nearest station.



**Fig. 2.** Displacement spectra for P (the left column) and S wave for the 2012 Sofia earthquake ( $M_w$  5.6).



**Fig. 3.** Displacement spectra for P (the left column) and S wave for five of the strongest aftershocks.

In table 1 are presented source parameters for the main shock and for 5 of the strongest aftershocks.

**Tab. 1.** Seismic source parameters of the 2012 seismic cluster that is located near the city of Pernik (Sofia seismogenic zone).

Date	$M_w$	Number of stations	$M_{0p}$ [N*m]	$\Delta\sigma_p$ [bar]	$r_p$ [km]	$M_{0s}$ [N*m]	$\Delta\sigma_s$ [bar]	$r_s$ [km]
22.05.2012	5.6	13	4.61E+17	52.99	3.53	2.13E+18	276.72	3.54
22.05.2012.	3.7	13	6.10E+14	18.95	0.53	4.68E+14	31.13	0.41
22.05.2012	4.5	11	1.57E+16	40.47	1.24	9.24E+16	21.73	2.36
22.05.2012	3.4	7	4.67E+13	52.28	0.17	5.50E+13	37.14	0.21
22.05.2012	4.0	13	2.08E+15	47.71	0.53	2.24E+15	44.95	0.61
14.07.2012	4.2	13	3.12E+15	74.48	0.61	5.44E+15	97.23	0.82

The results (based on P - wave and S - wave spectra) show that the stress drop for the main  $M_w$ 5.6 earthquake is about 53 bars for the P-wave and approximately 280 bars for the S - wave. The stress drop values estimated for P - waves are within the expected range for moderate earthquakes while for the S - waves are several times higher than the expected ones.

The stress drop average values for the aftershocks are between 19 bars and 97 bars. The aftershocks are characterized with lower stress drop values than those for the main event.

Source radius for the main shock is 3.5 km. Results for the aftershocks source radius are between 0.2 km and 2.4 km.

For aftershocks, seismic moments are in the range  $4.67 \times 10^{13} \leq M_0 \leq 9.24 \times 10^{16}$  (Nm) and the corresponding moment magnitudes,  $M_w$ , are from 3.0 to 4.6.

#### 4. Conclusions

- For 2012  $M_w$ 5.6 earthquake the stress drop (about 53 bars) estimated using P-waves is within the expected range for moderate earthquakes (from 10 to 100 bars), while for S-waves (approximately 280 bars) it is about 3 times higher than the expected one;
- The stress drop values estimated for P and S waves for aftershocks are within the expected range for moderate earthquakes. The stress drop values are between 19 bars and 97 bars.
- Source radius for the main shock is about 3.5 km. Estimates of the aftershock source radius are between 0.2 km and 2.4 km.

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## **Оценка огнищните параметри на земетресения, използвайки спектри на P и S-вълни—анализиран е сеизмичният кълъстер, реализиран през 2012 г. в сеизмогенна зона София**

Пл. Райкова

**Резюме:** Спектралният анализ на сеизмичните вълни е един от основните източници на информация за земетресенията. Представени са спектрите на P- и S-вълни за земетресения от сеизмичния кълъстер (главно събитие с магнитуд  $M_w$ 5.6 - афтершо-

ци) от 2012 г. Оценката на огнищните параметри е базирана на модела на Brune. Моделът се широко използван в сеизмологичните изследвания и е установено, че той е в добро съответствие с наблюденията от различни тектонски региони и е приложим за голям магнитуден диапазон. Проведеното изследване се основава на цифрови данни от Българската сеизмологична мрежа - NOTSSI (Национална оперативна телеметрична система за сеизмологична информация). Генерирани са спектри на преместване за P- и S- вълни, с цел да се определят някои от параметрите на сеизмичния източник, такива като сеизмичен момент, свалено напрежение и радиус. Определени са параметрите на 6 земетресения: на главното събитие с магнитуд по сеизмичен момент  $M_w$  5.6 и на пет афгършока с магнитуд  $M_w$  в интервала  $3.4 \div 4.5$ . За главното събитие стойностите на сваленото напрежение, изчислено за P - вълна, са в рамките на очаквания диапазон за умерено силно земетресение, докато за S – вълна стойностите на сваленото напрежение са няколко пъти по-високи от очакваните. Установено е, че афгършоците се характеризират с по-ниска стойност на сваленото напрежение в сравнение с основното събитие.