

## INVESTIGATION OF THE IMPACT OF ATMOSPHERIC REFRACTION ON PRECISION LEVELING MEASUREMENTS

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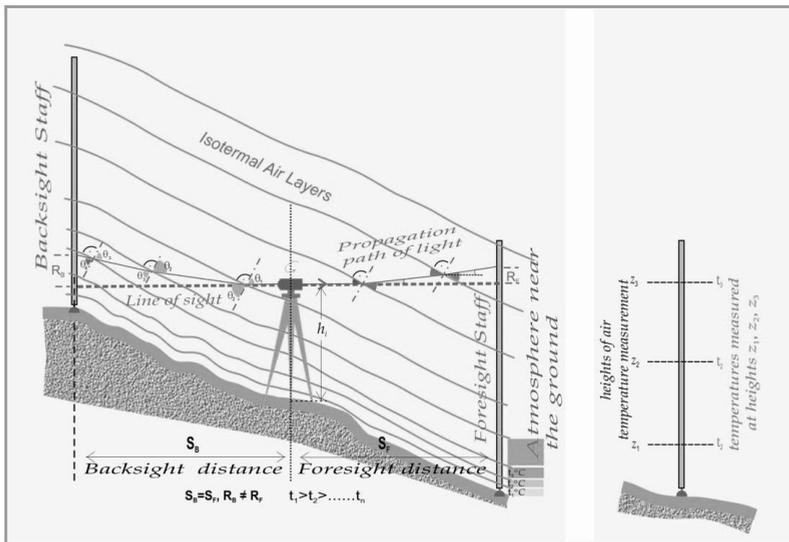
**Abstract.** The effect of atmospheric refraction on the results of precise leveling measurements is investigated. This study is based on level measurements for 30 km line provided by Geodesy, Cartography and Cadastre Agency. The results show that refraction's impact is not eliminated with one and the same positive and negative differences of elevation. The correction must be applied for each instrument set-up. Largest values of refraction are observed in the leveling distances with highest terrain slope. The error caused by refraction is a significant systematic error in the leveling measurements and it is mandatory to be applied.

**Key words:** precise leveling, atmospheric refraction, national leveling network

### Introduction

For the last three-four decades the precision of geodetic instruments has made tremendous progress. Modern electronic geodetic instruments allow a high degree of precision and automation but the accuracy and reliability of geodetic measurement is still strongly influenced by atmospheric conditions and by their knowledge. This phenomenon is particularly recognizable and still not fully unsolved in precise geodetic leveling. Atmospheric refraction is the deflection of light or other electromagnetic waves from the straight line due to the change in air density as a function of the height above the ground. As the geodetic measurements are carried out near the ground surface, the results are significantly influenced from the ground atmosphere. In geodetic leveling the horizontal line of sight passes through different isothermal layers of air (Fig. 1). This causes errors in readings on fore and back rods. The error caused by refraction is generally considered to be a significant systematic error in the leveling measurements. Already in the first half of the 20th century, prof. T. J. Kukkamaki (Finnish Geodetic Institute) investigated this phe-

nomenon and develop a mathematical model for correcting (reducing) its impact. He estimated a correction that is proportional to the difference in the measured two temperatures of air at heights of 0.5 m and 2.5 m. Initially only a few countries apply this correction, but now when it is known that it is necessary, the correction is widely used, especially in countries located in the middle and lower latitudes.



**Fig. 1.** Refraction

## Theoretical model

The refraction correction for geodetic leveling must be applied to each single set-up (Kukkamaki T. J., 1939):

$$R = -2 \times 10^{-6} \alpha \left( \frac{S}{50} \right) \Delta t \Delta h [m], \quad (1)$$

where:  $S$  is the average value of the distances between rods in meters (section length);  $Dt = t_3 - t_1$  is the temperature difference ( $^{\circ}\text{C}$ ), calculated from the measured temperatures at two heights, for example:  $t_3$  – at a height of 2.5 m and  $t_1$  – at a height of 0.5 m (Fig. 1);  $Dh$  – measured difference of elevation in set-up (in meters);  $a$  – is a function, dependent on an assumed temperature function:

$$T = a + bz^c, \quad (2)$$

where:  $T$  ( $^{\circ}\text{C}$ ) is a temperature at a height  $z$  above the ground surface, when  $z$  is less than 300cm;  $a, b$  и  $c$  are constants for any instant and vary with time.

A is sometimes assumed constant (NOAA, 1981), but is rigorously calculated as:

$$\alpha = 5 \cdot \frac{95}{z_3^c - z_1^c} \left[ \frac{1}{c+1} (L_F^{c+1} - L_2^{c+1}) - h_i (L_F - L_B) \right], \quad (3)$$

where:  $z_3$  и  $z_1$  are heights of the measurement of air temperature;  $L_F$  и  $L_B$  are heights of the line of sight on the fore and back rods, respectively;  $h_i$  is instrument height, and  $c$  is an exponent (Kukkamaki T. J., 1939).

The temperature model (2) and corresponding refraction correction are based on the following assumptions: the refraction coefficient of air depends mainly on temperature; the effect of humidity is negligibly small for optical propagation; isothermal surfaces are parallel to the ground; the ground slope beneath the sightline is uniform in a single set-up of the instrument.

Exponent  $c$  can be calculated using temperatures measured by three temperature sensors, located at different heights  $z_1, z_2, z_3$  (Fig. 1) arranged such that  $z_1/z_2 = z_2/z_3$ . For each measured temperature are drawn three equations of the type of (3). Through the transformations the estimation of the exponent  $c$  required to obtain the coefficient  $\alpha$  (1), (2) is reached:

$$c = \frac{\ln \left( \frac{\Delta t_2}{\Delta t_1} \right)}{\ln \left( \frac{z_2}{z_1} \right)}. \quad (4)$$

Due to the large air temperature fluctuations direct temperature gradient determination should be performed at every single set-up at the same time as the levelling measurements.

## Data processing

For the purpose of this study the level measurements provided by Bulgarian Geodesy, Cartography and Cadastre Agency were used. The measurements were made with a precise electronic digital/barcode levelling system Sokkia SDL 1X with a couple of invar rods. Simultaneously with the leveling, the air temperatures were measured for each set-up at heights of 0.5 m, 1.5 m and 2.5 m. Digital thermometers are used. Their sensors are attached to the back side of the rods and are protected from direct sunlight.

The temperature differences are calculated with the average values of the temperatures measured on the two rods at 2.5m and 0.5m, respectively. Leveling book for one leveling distance is shown at Table 1. The measured temperatures are checked in order to be acceptable, the temperature differences between the upper and lower thermometer of the rods should be between  $-3.0^\circ\text{C}$  and  $+1.0^\circ\text{C}$ . Also the difference between the temperature differences of two successive setups should be between  $-3.0^\circ\text{C}$  and  $+3.0^\circ\text{C}$ . When the

temperature measurements meet this requirements the average of temperature differences between upper and lower thermometer in back and for rode are used. In several leveling distances one of the thermometers has failed and did not meet the requirements. In this case only temperature differences measured on one rode is used. The refraction correction is calculated for each set-up (Table 1, column 7) and is aggregated for the whole distance and  $a = 70$  (Hytonen E.,1967; NOAA, 1981). The measured temperatures are given in Table 2.

**Table 1.** Levelling book

LEVELING BOOK									
Leveling line		№ 47 - KHP Kazanluk BHP № 86 Haskovo							
Leveling distance		HP1-HP2							
Date	20 may 2016	Observer		.....					
Start	11 h 50 min	Instrument		Sokkia SDL 1X					
End	h min	№		123456					
Destination	north-south	Rod 1		67147					
		Rod 2		67148					
Wheater conditions		clear view		weak wind		sunny			
№	Dist. [m]	reads		Difference (m)		d2 (MM)	R (MM)	diff. (MM)	ΔH (M)
		back 1 fore 2	back 2 fore 2	(back 1) -(fore 1)	(back 2) -(fore 2)				
1	2	3	4	5	6	7	8	9	10
HP1	20.76	0.74420	0.74422						
1	23.38	1.62199	1.62195	-0.87779	-0.87773	0.013	0.005	-0.06	-0.87776
1	26.77	1.21429	1.21429						
2	27.31	1.58203	1.58200	-0.36774	-0.36771	0.043	0.005	-0.03	-0.36773
2	28.65	1.32574	1.32585						
3	28.43	1.52577	1.52579	-0.20003	-0.19994	0.001	0.002	-0.09	-0.19999
3	27.75	1.29577	1.29568						
4	28.41	1.57473	1.57471	-0.27896	-0.27903	-0.002	0.003	0.07	-0.27900
4	29.11	1.36703	1.36701						
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9	27.86	1.29703	1.29698						
10	29.69	1.79338	1.79336	-0.49635	-0.49638	-0.044	0.005	0.03	-0.49637
10	29.72	1.36363	1.36364						
11	29.61	1.95155	1.95161	-0.58792	-0.58797	0.002	0.004	0.05	-0.58795
11	28.30	1.27017	1.27023						
12	25.91	1.72158	1.72166	-0.45141	-0.45143	-0.038	0.006	0.02	-0.45142
12	29.22	1.36455	1.36449						
13	27.99	1.64569	1.64564	-0.28114	-0.28115	0.027	0.002	0.01	-0.28115
13	27.49	1.20420	1.20411						
HP2	13.40	0.95056	0.95051	0.25364	0.25360	0.050	0.001	0.04	0.25362
		SF [m]		∑ΔH1 [m]	∑ΔH2 [m]	∑ d2[m]	∑R [m]	∑ [mm]	
		22.75835		-5.49841	-5.49841	0.00011	0.00006	0.000	
S =	0.764 km			d =	0.000 mm				
Ter	20.6 C°			ΔHm =	-5.49841 m				
Lcp =	1000 mm			d <sub>1</sub> =	-0.00004 m				
a =	0.000002			d <sub>2</sub> =	0.00011 m				
				SR =	0.00006 m				
				ΔHcorr =	-5.49853 m				

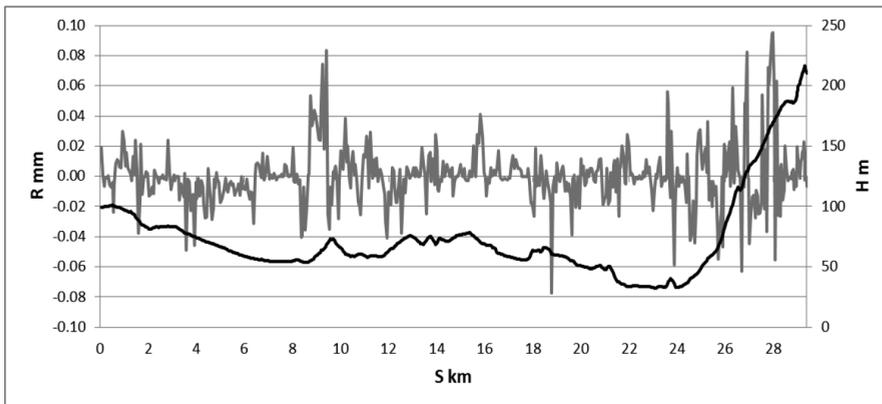
**Table 2.** Meteorological book

METEOROLOGICAL BOOK									
Leveling distance		HP1-HP2							
Date		20 may 2016							
st. №	rod	Heights of meas.			mean		Temperatures		
	back	0.5 M	1.5 M	2.5 M	back	fore	Tb-Tet	Tb-Tf	T3-T1
	fore	T1	T2	T3	Tb	Tf			
1	2	3	4	5	6	7	8	9	10
1	rod №1	23.5	23.4	23.4	23.4		2.8		-0.2
	rod №2	23.4	23.3	23.1		23.3		-0.2	
2	rod №2	23.9	23.8	23.7	23.8		3.2		-0.3
	rod №1	23.1	22.9	22.7		22.9		-0.9	
3	rod №1	23.5	23.4	23.3	23.4		2.8		-0.2
	rod №2	23.5	23.4	23.3		23.4		0.0	
4	rod №2	23.2	23.0	22.9	23.0		2.4		-0.3
	rod №1	23.2	23.1	23.0		23.1		0.1	
5	rod №1	22.5	22.4	22.2	22.4		1.8		-0.3
	rod №2	22.8	22.7	22.6		22.7		0.3	
6	rod №2	22.9	22.8	22.7	22.8		2.2		-0.2
	rod №1	22.3	22.2	22.1		22.2		-0.6	
7	rod №1	23.4	23.3	23.1	23.3		2.7		-0.2
	rod №2	24.0	23.9	23.8		23.9		0.6	
8	rod №2	24.5	24.3	24.2	24.3		3.7		-0.4
	rod №1	24.0	23.8	23.6		23.8		-0.5	
9	rod №1	23.6	23.4	23.3	23.4		2.8		-0.3
	rod №2	22.8	22.7	22.6		22.7		-0.7	
10	rod №2	22.5	22.4	22.4	22.4		1.8		-0.2
	rod №1	23.6	23.4	23.3		23.4		1.0	
11	rod №1	23.5	23.4	23.3	23.4		2.8		-0.1
	rod №2	23.5	23.4	23.4		23.4		0.0	
12	rod №2	23.7	23.5	23.3	23.5		2.9		-0.3
	rod №1	24.5	24.4	24.3		24.4		0.9	
13	rod №1	23.1	23.1	23.0	23.1		2.5		-0.2
	rod №2	22.6	22.5	22.4		22.5		-0.6	
14	rod №2	23.8	23.6	23.4	23.6		3.0		-0.3
	rod №1	22.6	22.5	22.5		22.5		-1.1	

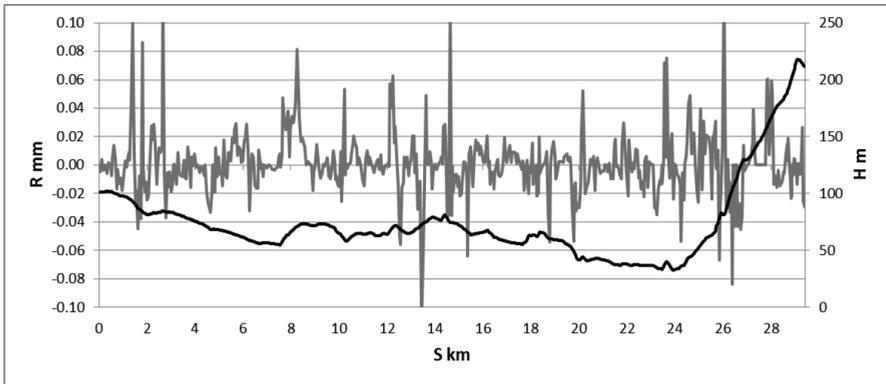
## Analysis

An adjustment of a leveling line was performed. The leveling line consists of 22 leveling distances. For the purposes of this study a section (part of whole leveling line) was analysed. The section lies between two Fundamental benchmarks and is 30 km long. On Fig. 2 and Fig. 3 the values of refraction correction for every set-up along the line in fore and back leveling are shown along with cross section of the leveling line. The values of refraction correction for the whole length of the line is 0.6 mm. On Fig. 4 and Fig. 5 the refraction correction values for every levelling distance are given for fore and back levelling respectively. Refraction correction, calculated for each leveling distance, varies from 0 mm to 1.5 mm. The largest values of the refraction correction are observed in the leveling distances with highest terrain slope. In case of leveling successive distances with positive and negative difference of elevation, the refractive error is not compensated. For example, in the segment between the benchmark at 6.8 km and the benchmark at 19.7 km the value of refraction correction is 0.6 mm, although the difference of elevation between the endpoints is almost zero, and line going through sequential climb and descent.

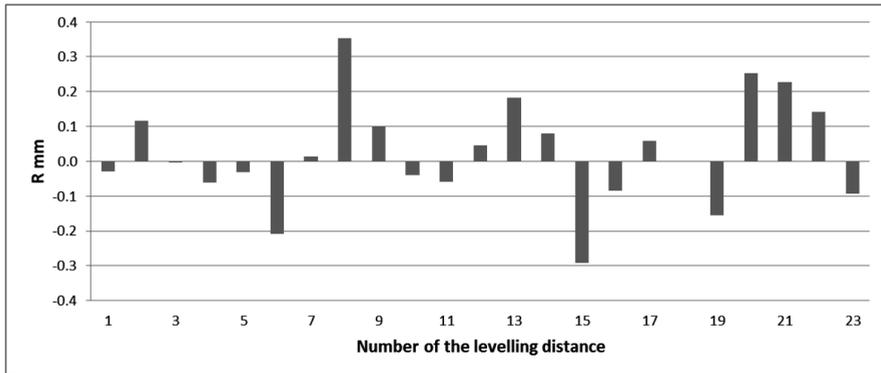
The precision of measured temperatures give a significant impact on the value of the systematic refractive error (correction respectively). Numerous meteorological publications show that in the night the ground is colder than the air just above it. Soon after the sunrise temperature of the air is decreasing with the height and the temperature of the ground becomes higher than the temperature of the air just above it. For this reason the temperature gradient is negative at day and positive at night. The absolute values of the vertical gradient are greater in the clear sky, day or night (Kukkamaki T. J., 1978).



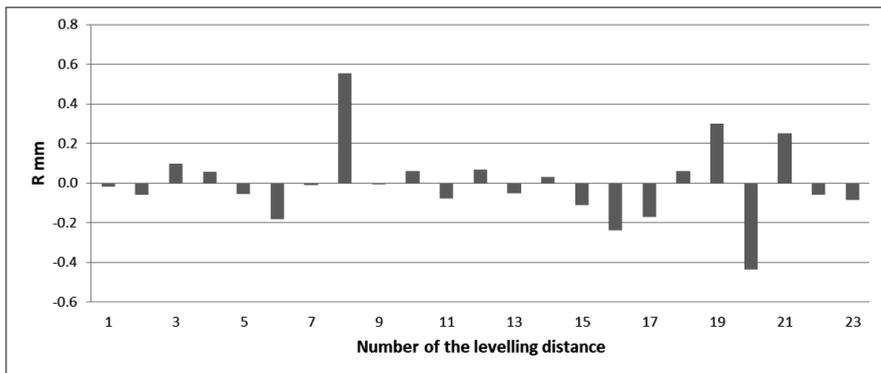
**Fig. 2.** Refraction correction calculated for every set-up in fore leveling in grey and cross section of the leveling line in black.



**Fig. 3.** Refraction correction calculated for every set-up in back leveling in grey and cross section of the leveling line in black



**Fig. 4.** Refraction correction values for every leveling distance in fore leveling



**Fig. 5.** Refraction correction values for every leveling distance in back leveling.

Measurements of the temperature should be done simultaneously with leveling by aspiration thermometers (with forced air flow) to obtain maximum reliable temperature gradient values. Thermometer readings should be monitored and evaluated. In order to be acceptable, the temperature differences between the upper and lower thermometer of the rods should be between  $-3.0^{\circ}\text{C}$  and  $+1.0^{\circ}\text{C}$ . Also the difference between the temperature differences of two successive setups should be between  $-3.0^{\circ}\text{C}$  и  $+3.0^{\circ}\text{C}$  (NOAA, 1999).

In case of slope, the refraction is greater on sight close to surface, so the rod reading on that sight that is close to the terrain is more affected than other. This effect is more noticeable on long gentle slopes, when long sight lines are used (Angus-Leppan P.V., 1884).

If the measured temperatures are outside these limits, it is recommended not to conduct leveling measurements until the cause is eliminated or the weather conditions are improved. In processing and analyzing measurements in the test section, the temperatures that do not meet the above conditions are excluded from processing. Instead, only the thermometers of one rod are used or temperatures are interpolated from previous and next set-up.

## **Conclusions**

The results show that the refraction correction is commensurable with the correction for the difference between the average of the rod meter and the reference one. This correction must be applied for each set-up and it is not eliminated with the same positive and negative differences of elevation. The largest values of refraction correction are observed in the leveling distances with highest terrain slope. Refraction error will eliminate almost exactly if back-sight and foresight are well balanced and the terrain is flat.

Measurements of the temperatures should be done with aspiration thermometers, simultaneously with measurements of the leveling. Incorrectly measured temperatures have a negative impact and they can lead to wrong calculated correction and contaminated final results.

Applying this correction does not eliminate error caused by the atmospheric refraction, but it can be reduced by applying various measuring procedures associated with balancing of the length of sights, limiting the length of sight, not reading the portion of level staff close to ground and choice proper weather conditions.

It is recommended to perform experimental research and develop a model for the vertical refraction which is suitable for the territory of Bulgaria.

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## Изследване на влиянието на атмосферната рефракция върху прецизните нивелачни измервания

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**Резюме:** Изследвани са някои проблеми при прилагане на корекцията заради вертикалната рефракция в приземния въздушен слой към измерените превишения при прецизна нивелация. Получените резултати показват, че корекцията трябва да се въвежда за всяка станция и не се нулира при преминаване на едни и същи положителни и отрицателни превишения. Неточно измерените температури имат негативно влияние, могат до доведат до грешно изчислена корекция и да повлияят на крайните резултати. По тази причина е важно температурите да се измерват едновременно с нивелацията, посредством аспирационен термометър, с точност не по-малка от  $\pm 0.1^\circ\text{C}$ . Препоръчително е да се направят експериментални изследвания и да се приеме подходящ за територията на България модел за отчитане на вертикалната рефракция.