APPROXIMATION OF INCOMPRESSIBLE FLUID TO THE GEODYNAMO CONVECTION

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Abstract. The problem of applying the Boussinesq approximation to the convection in the Earth's core is discussed. It is shown that the Boussinesq approximation neglects the essential part of the heat transport in the core, the adiabatic heat flux, which does not vanish in the incompressible limit. It also neglects the cooling due to the work of the Archimedean force. As a result, the law of energy conservation reduces to the heat conservation only. Thus, the Boussinesq approximation is inadequate for describing of the Earth core convection. Therefore we propose here a new, Incompressible approach, which takes all these effects into account. In the frame of this new approach we estimate the "natural" units for the Earth's core convection. On this base we redefine the value of the Rayleigh number which is widely discussed in literature.

Key words: anelastic convection, Boussinesq approximation, geodynamo, compressibility.

Introduction

Generation of Earth's magnetic field is energetically supported by the convection in the Earth's liquid core. That is why the adequate describing of the convection is so important for the geodynamo.

Most of the computer simulations in this area (see e.g. Fearn and Morrison (2001) Sakuraba, and Kono (1999) Cupal et al (2004)), were carried out in the Boussinesq approximation. Even the "numerical dynamo benchmark", Christensen et al (2001), works on this base. Therefore, the question whether the Boussinesq approximation is adequate for describing of the Earth's core convection is very significant for the geodynamo problem. The present work is devoted to answer this question.

The main problem of theoretical physics is understanding the sense of the physical phenomena. This understanding is usually impossible without simplification of the model. That is why, the neglecting the compressibility of the Earth's core in the Boussinesq approximation, is an useful step for understanding the convection in the core, in spite of the relatively big density difference ($\Delta\bar{\rho}/\bar{\rho} \sim 20\%$) between its bottom, $r = r_1$, and top, $r = r_2$, boundaries. For the first and for the second ones, abbreviations ICB (Inner Core Boundary) and CMB (Core Mantle Boundary) are commonly used.

The reference state in the Earth's core is adopted to be hydrostatic, well mixed and, consequently, adiabatic: $\bar{T}(r) = \bar{T}(r_1)\left(\bar{\rho}(r)/\bar{\rho}(r_1)\right)^{\gamma}$. Respectively, the temperature difference $\Delta\bar{T}$ between both boundaries also vanishes in the inviscid limit $\Delta\bar{\rho} \to 0$. Therefore, we accept both values to be constant in this incompressible limit: $\bar{\rho} = \bar{\rho}_o$ and $\bar{T} = \bar{T}_o$.

We will see in the next section that in spite of $\Delta\bar{T} \to 0$, the adiabatic heat flux $\mathcal{H} = 4\pi\rho_o\kappa(d\bar{T}/dr)$ does not vanish in the incompressible limit. There is also one more value, the gravitational acceleration $g = g_o(r/r_o)$ which does not vanish in this limit. It rises only 2.8 times between the two boundaries and the Boussinesq approximation takes this change into consideration. However, the adiabatic heat flux which enhances 23 times is neglected in BA! May be its value is too small to be taken into account, in spite of being 23 times greater? On the opposite, the adiabatic heat flux is equal to approximately 2/3 of the whole heat flux leaving the core on CMB. Neglecting such a large effect is a besetting sin of the Boussinesq approximation. Therefore, here we propose a new, Incompressible Approach, which takes the adiabatic heat flux into account.

Incompressible reference state

Our aim here is not to study the whole reference state of the Earth's core, but only to watch its behavior in the incompressible limit $\Delta\bar{\rho} \to 0$. Our estimations below are based on the Preliminary Reference Earth Model (PREM) by Dziewonski and Anderson (1981).

The equations which govern the gravitational acceleration $\bar{g}$, temperature $\bar{T}$, and density $\bar{\rho}$ for our adiabatic reference state are

$$\frac{1}{r^2}\frac{dr^2\bar{g}}{dr} = 4\pi G\bar{g}, \qquad \frac{1}{\bar{T}}\frac{d\bar{T}}{dr} = -\frac{1}{r^2}\frac{\alpha\bar{g}}{c_p}, \qquad \frac{d\bar{\rho}}{dr} = -\frac{1}{\gamma}\frac{\alpha\bar{g}}{c_p}. \qquad (1.1,2,3)$$

Here G is the gravitational constant, $\alpha = -\bar{\rho}(\partial\bar{\rho}/\partial\bar{T})_p$ is the coefficient of thermal expansion, γ is the Grüneisen parameter and c_p is the specific heat at constant pressure. The values of these and other parameters used in this paper are given in Table 1.

Table 1

G	$6.67\times10^{-11} m^3 kg^{-1} s^{-2}$	α	$1.4\times10^{-5} K^{-1}$	c_p	$860\, m^2 s^{-2} K^{-1}$
r_1	$1.22\times10^{6} m$	r_2	$3.48\times10^{6} m$	d	$2.26\times10^{6} m$
κ	$5\times10^{-5} m^2 s^{-1}$	η	$2 m^2 s^{-1}$	κ_T	$2 m^2 s^{-1}$
$\bar{\rho}_o$	$1.1\times10^{4} kg\, m^{-3}$	$\bar{T}_o$	$4650 K$	γ	1.35

Here $d = r_2 - r_1$ is the thickness of the liquid core spherical shell, η is the magnetic diffusivity, κ is the thermal diffusivity and κ_T is its turbulent value. Values of $\overline{\rho}_o$ and $\overline{T}_o$ are defined by relations: $\overline{\rho}_o = (\overline{\rho}_{ICB} + \overline{\rho}_{CMB})/2$ and $\overline{T}_o = (\overline{T}_{ICB} + \overline{T}_{CMB})/2$. We take these parameters from Roberts and Glatzmaier's (2000) work.

Crude estimation based on (1.2,3) gives the difference of temperature and density between ICB and CMB:

$$\Delta\overline{T} \sim \overline{T}_o D, \qquad \Delta\overline{\rho} \sim \overline{\rho}_o D, \quad \text{where } D = \frac{g_o \alpha d}{c_p} = 0.27 \quad \text{and } g = 7.2 ms^{-2}. \tag{1.4}$$

This estimate shows that $\overline{T}$ and $\overline{\rho}$ change slowly ($\sim D$) with r . At the same time the variable $g(r)$ changes approximately three times between ICB and CMB. So the variables of RS can be divided into two groups. The first one includes parameters such as $\overline{T}$, and $\overline{\rho}$ which change slowly between r_1 and r_2 . The second ("fast") group consists of parameters such as e.g. $\overline{g}$, which change essentially between these two boundaries.

The solution of the equations (1.1-3) can be searched by iterations. In the leading approximation we will neglect the change of the "slow" parameters: $\overline{\rho} = const\ (= \overline{\rho}_o)$ and $\overline{T} = const\ (= \overline{T}_o)$ and will take into consideration the variation of the "fast" values only. In the next approximation we will use the fast approximation values in order to obtain the small variation of the "slow" variables. This process can be repeated for the next approximations, but further we will be interested mainly in the leading approximation which could be called the incompressible one.

The fast variables are given by the right hand sides of the equations (1.1-4). They are $\overline{g}(r)$, $D(r)$, $d\overline{T}/dr$, and $d\overline{\rho}/dr$. An additional extra-fast variable, the conductive adiabatic heat flux

$$\mathcal{H} = 4\pi r^2 \overline{\rho}_o \kappa c_p \frac{d\overline{T}}{dr}$$

has to be included in this group as well. Then the direct integration of (1.1-3) gives:

$$\overline{g} = \overline{g}_o \frac{r}{r_o}, \quad \overline{D} = D_o \frac{r}{r_o}, \quad \frac{d\overline{T}}{dr} = \frac{\overline{T}_o}{r_o} D_o \frac{r}{r_o}, \quad \frac{d\overline{\rho}}{dr} = -\frac{\overline{\rho}_o}{r_o} \frac{D_o}{\gamma} \frac{r}{r_o}, \quad \mathcal{H} = \frac{r^3}{r_o^3} \mathcal{H}_o, \tag{1.6}$$

where $r_o = (r_1 + r_2)/2 = 2.35 \times 10^6 m$ and

$$\overline{g} = \frac{4\pi r_o G \overline{\rho}_o}{3} = 7.22\ ms^{-2}, \quad D_o = \frac{\overline{g}_o \alpha r_o}{c_p} = 0.276, \quad \mathcal{H}_o = 4\pi r_o^2 \overline{\rho}_o \kappa \alpha \overline{g}_o \overline{T}_o = 1.8\ TW. \tag{1.7}$$

The results of our Incompressible reference state (IRS) are compared in Table 2 with the values used by Roberts and Glatzmaier (2000) (RG).

Table 2

	$\bar{g}(r_1)$	$\bar{g}(r_2)$	$\mathcal{H}(r_1)$	$\mathcal{H}(r_2)$
IRS	$3.8\,ms^{-2}$	$10.7ms^{-2}$	$0.25TW$	$5.8TW$
RG	$4.4\,ms^{-2}$	$10.7ms^{-2}$	$0.3TW$	$5.4TW$

This Table shows that incompressibility of the reference state does not disturb it essentially. For any variable A, let us introduce a value $DA = \Delta A / A(r_o)$, where $\Delta A = A(r_2) - A(r_1)$. Then it follows from (1.6) that

$$D(\bar{g}) = \frac{d}{r_o} = 0.96, \quad D(\mathcal{H}) = \left(1 + \frac{d}{2r_o}\right)^3 - \left(1 - \frac{d}{2r_o}\right)^3 = 3.1. \tag{1.8}$$

Equations for the "slow" variables can be obtained by substitution of the "incompressible" D into (1.2,3):

$$\frac{d\overline{T^{(1)}}}{dr} = \frac{\bar{T}_o}{r_o} D_o \frac{r}{r_o}, \qquad \frac{d\overline{\rho^{(1)}}}{dr} = -\frac{\bar{\rho}_o}{r_o}\frac{D_o}{\gamma}\frac{r}{r_o}.$$

We are interested not in the solutions of these equations but only in their differences on both boundaries:

$$\Delta\bar{T} = \bar{T}(r_1) - \bar{T}(r_2) = \bar{T}_o D_o \frac{r_2^2 - r_1^2}{2r_o} = \bar{T}_o D_o \frac{d}{r_o} \sim \bar{T}_o D_o, \qquad \Delta\bar{\rho} = \bar{\rho}_o \frac{D_o}{\gamma}.$$

It follows from here that

$$D(\bar{T}) = \frac{\Delta\bar{T}}{\bar{T}_o} = D_o \frac{d}{r_o} = 0.26, \quad D(\bar{\rho}) = \frac{\Delta\bar{\rho}}{\bar{\rho}_o} = \frac{D_o}{\gamma}\frac{d}{r_o} = \frac{D(\bar{T})}{\gamma} = 0.19. \tag{1.9}$$

Parameter D_o characterizes the compressibility of the liquid in the Earth's core. In the incompressible limit $D_o \to 0$ the differences of the temperature and density between the top and the bottom boundaries vanish. However the "fast" parameters $g(r)$ and $\mathcal{H}(r)$ are unaffected by this limit and respectively cannot be neglected, if parameter d does not change. In fact, neglecting the adiabatic heat flux leads to disturbance of the law of the energy conservation: the energy flux enhances without energy sources.

That is why the **commonly used Boussinesq approximation is incorrect in application to the Earth's core: it violates the energy conservation law**. This approximation could be correct if the change of the adiabatic heat flux is negligible i.e. if $D(\mathcal{H}) \ll 1$. As (1.8) shows, the above is possible if $3d/2r_o \ll 1$, say $d = (2/3)r_o/10 \sim 150km$. In this case we must neglect the changes not only in the "slow" but in the "fast" parameters as well. This means that $\bar{\mathbf{g}}$ in the correct Boussinesq approximation does not depend on r.

The incompressible approach to the Earth's core convection

Anufriev et al (2005) proposed an anelastic liquid approximation for the Earth's core convection. Their momentum heat transport and induction equations can be written in a form:

$$\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}=\nabla\frac{P}{\bar{\rho}(r)}+\mathbf{1}_r\bar{g}(r)\alpha\vartheta+\frac{\nabla\times\mathbf{B}\times\mathbf{B}}{\bar{\rho}(r)\mu_o}-2\mathbf{\Omega}\times\mathbf{V}+\frac{\mathbf{F}^{\nu}}{\bar{\rho}(r)},\qquad \nabla\cdot\bar{\rho}(r)\mathbf{V}=0, \qquad (2.1,2)$$

$$\frac{\partial \vartheta}{\partial t}+(\mathbf{V}\cdot\nabla)\vartheta=\frac{1}{\bar{\rho}(r)}\nabla\cdot\bar{\rho}(r)[\kappa_T\nabla\vartheta+\kappa\nabla\bar{T}]-\frac{\bar{g}(r)\alpha}{c_p}\left[\vartheta V_r-\kappa_T\frac{\partial\vartheta}{\partial r}\right]+\frac{H}{c_p}, \qquad (2.3)$$

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times[\mathbf{V}\times\mathbf{B}]+\eta\nabla^2\mathbf{B},\qquad \nabla\cdot\mathbf{B}=0, \qquad (2.4,5)$$

where $\mathbf{F}^{\nu}$ is the viscous force.

The Earth's core convection is driven by the heat extracted at ICB due to solidification of iron on the inner core surface. The simplest model of this process is the uniform super-adiabatic heat flux $Q_{sa}(r_1)$ prescribed at ICB. To complete the formulation of the problem we also assume that the super-adiabatic temperature is uniform at CMB:

$$\frac{\partial\vartheta(r_1)}{\partial r}=\frac{Q_{sa}}{4\pi r_1^2 k_T},\qquad \vartheta(r_2)=0, \qquad (2.6,7)$$

where $k_T=\bar{\rho}_o c_p\kappa_T$ is the thermal conductivity.

As usual, we will use the no-slip boundary conditions for the flow velocity and correspondent conditions for the magnetic field at both boundaries. Further we will neglect the compressibility and use the Incompressible reference state (IRS) (1.6,7) of the Earth's core. Then the momentum (2.1), the continuity (2.2) and the heat transport (2.3) equations take their incompressible form:

$$\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}=\nabla\frac{P}{\bar{\rho}_o}+\mathbf{1}_r\bar{g}(r)\alpha\vartheta+\frac{\nabla\times\mathbf{B}\times\mathbf{B}}{\bar{\rho}_o\mu_o}-2\mathbf{\Omega}\times\mathbf{V}+\nu_T\nabla^2\mathbf{V},\qquad \nabla\cdot\mathbf{V}=0, \qquad (2.10,11)$$

$$\frac{\partial \vartheta}{\partial t}+(\mathbf{V}\cdot\nabla)\vartheta=\nabla\cdot[\kappa_T\nabla\vartheta+\kappa\nabla\bar{T}]-\frac{\bar{g}(r)\alpha}{c_p}\vartheta V_r+\frac{H}{c_p}. \qquad (2.12)$$

The last of them differs from the heat transport equation of the Boussinesq approximation by two additional terms. The first one $\kappa\nabla^2\bar{T}(r)$ describes the heat loss due to supporting the adiabatic heat flux. We will refer to it as the adiabatic cooling. The other new term, $(D(r)/d)\vartheta V_r$, describes the cooling due to the work of the Archimedean force $\mathbf{1}_r\bar{g}(r)\alpha\vartheta(\mathbf{r})$ in the momentum equation. We will call it the Archimedean cooling. The fourth term in rhs of (2.3) describes the cooling due to the conversion of the internal energy into the kinetic energy of the non-resolved turbulent eddies. As this term is small, we neglect it in our approximation.

It follows from (1.6) that the adiabatic heat flux is linear in r and thus the adiabatic cooling is uniform:

$$\kappa\nabla\overline{T}(r) = -\mathbf{1}_{r\kappa}\kappa\frac{D_o\overline{T}_o}{r_o}\frac{r}{r_o}, \qquad \nabla\cdot\kappa\nabla\overline{T}(r) = -3\kappa D_o\frac{\overline{T}_o}{r_o^{\,2}}.$$

Induction equation (2.4) is a consequence of the Maxwell and Ohm equations

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\mathbf{E}, \quad \text{where } \mathbf{E} = \frac{\mathbf{j}}{\sigma} - \mathbf{V}\times\mathbf{B} \quad \text{and} \quad \mu_o\mathbf{j} = \nabla\times\mathbf{B}. \tag{2.13}$$

Mltiplying (2.7) by $\overline{\rho}_o\mathbf{V}$, (2.9) by $c_p\overline{\rho}_o$ and (2.13) by $\mathbf{B}/\mu_o$ we obtain the energy balance for the kinetic, $\varepsilon^k = \overline{\rho}_o\mathbf{V}^2/2$,

$$\frac{\partial\varepsilon^k}{\partial t} = -\nabla\cdot[\mathbf{V}\varepsilon^k + P\mathbf{V} + \overline{\rho}_o\nu\mathbf{V}\times\nabla\times\mathbf{V}] + \overline{\rho}_o g V_r\alpha\vartheta + \mathbf{V}\cdot\mathbf{j}\times\mathbf{B} - \overline{\rho}_o\nu[\nabla\times\mathbf{V}]^2, \tag{2.14}$$

for the internal, $\varepsilon^\vartheta = c_p\overline{\rho}_o\vartheta$,

$$\frac{\partial\varepsilon^\vartheta}{\partial t} = -\nabla\cdot\left[\mathbf{V}\varepsilon^\vartheta - \overline{\rho}_o c_p\left(\kappa_T\nabla\cdot\vartheta + \mathbf{1}_r\kappa\frac{\partial\overline{T}}{\partial r}\right)\right] + \overline{\rho}_o g\alpha\vartheta V_r, \tag{2.15}$$

and for the magnetic, $\varepsilon^m = \overline{\rho}_o\mathbf{B}^2/2\mu_o$,

$$\frac{\partial\varepsilon^m}{\partial t} = -\nabla\cdot\frac{\mathbf{B}\times\mathbf{E}}{\mu_o} - \mathbf{V}\cdot\mathbf{j}\times\mathbf{B} - \eta\mu_o\mathbf{j}^2, \qquad \mathbf{E}=\eta\mu_o\mathbf{j} - \mathbf{V}\times\mathbf{B} \tag{2.16}$$

energies.

Summering (2.14), (2.15) and (2.16) we obtain the equation for the whole energy balance:

$$\frac{\partial}{\partial t}[\varepsilon^k + \varepsilon^\vartheta + \varepsilon^m] = -\nabla\cdot\mathbf{I} + \overline{\rho}_o H - \eta\mu_o\mathbf{j}^2 - \overline{\rho}_o\nu[\nabla\times\mathbf{V}]^2, \tag{2.17}$$

where

$$\mathbf{I} = \mathbf{V}[\varepsilon^k + P + \varepsilon^\vartheta] + \nabla\cdot\frac{\mathbf{B}\times\mathbf{E}}{\mu_o} - \overline{\rho}_o\nu\mathbf{V}\times[\nabla\times\mathbf{V}] - \overline{\rho}_o c_p\left(\kappa_T\nabla\cdot\vartheta + \mathbf{1}_r\kappa\frac{\partial\overline{T}}{\partial r}\right), \tag{2.18}$$

is the energy flux.

Further we will assume that the internal heating is absent and as a result the whole energy conserves. This means that changing the energy in any volume has to be due only to the flux across its walls. Then it follows from (2.17) that in the absence of radioactive heating, the heating in the volume is defined only by the Ohmic and the viscous dissipations:

$$\overline{\rho}_o H = \eta\mu_o\mathbf{j}^2 + \overline{\rho}_o\nu[\nabla\times\mathbf{V}]^2. \tag{2.19}$$

The viscous dissipation is small and will be neglected everywhere.

Let us integrate (2.17) over the whole liquid core. Then we obtain:

$$\frac{\partial}{\partial t}[E^k + E^\vartheta + E^m] = \mathcal{I}(r_2) - \mathcal{I}(r_1), \tag{2.20}$$

where

$$E^k = 4\pi \int_{r_1}^{r_2} \left\langle \varepsilon^k \right\rangle r^2 dr, \quad E^\vartheta = 4\pi \int_{r_1}^{r_2} \left\langle \varepsilon^\vartheta \right\rangle r^2 dr, \quad E^m = 4\pi \int_{r_1}^{r_2} \left\langle \varepsilon^m \right\rangle r^2 dr, \tag{2.21}$$

means the whole kinetic, internal and magnetic energies and

$$\mathcal{I}(r) = 4\pi r^2 \left\langle I_r(r) \right\rangle. \tag{2.22}$$

means the whole energy flux. The angle brackets here mean averaging over whole sphere:

$$\left\langle A \right\rangle(r) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} A(r,\vartheta,\phi) \sin\vartheta d\vartheta d\phi. \tag{2.23}$$

The normal component of the flow velocity vanishes on the solid boundaries. So the energy flux (2.18) averaging over sphere takes a form:

$$\left\langle I_r(r_{1,2}) \right\rangle = \frac{\mathbf{1}_r \cdot \left\langle \mathbf{B} \times \mathbf{E} \right\rangle (r_{1,2})}{\mu_o} - \bar{\rho}_o \nu \mathbf{1}_r \cdot \left\langle \mathbf{V} \times [\nabla \times \mathbf{V}] \right\rangle (r_{1,2}) - \bar{\rho}_o c_p \frac{\partial}{\partial r} \left\langle \kappa_T \vartheta + \mathbf{1}_r \kappa \bar{T} \right\rangle (r_{1,2}). \tag{2.24}$$

The whole energy $E^k + E^\vartheta + E^m$ is limited. So the lhs of (2.20) vanishes under averaging it over a long time interval. Then this equation takes the form of balance of the energy fluxes on ICB and CMB: $\mathcal{I}(r_2) - \mathcal{I}(r_1) = 0$. The first and the second terms in (2.24) describe the fluxes of the kinetic and magnetic energies. They change the kinetic energies of the mantle and the inner core and the magnetic energy into the whole space. These terms also vanish under averaging over the long time interval. Then the energy flux balance converts into the balance of the adiabatic and super-adiabatic heat fluxes which can be written in a form

$$k_T r_1^2 \frac{\partial \left\langle \vartheta \right\rangle (r_1)}{\partial r} - k_T r_2^2 \frac{\partial \left\langle \vartheta \right\rangle (r_2)}{\partial r} = k \left[r_2^2 \frac{\partial \bar{T}(r_2)}{\partial r} - r_1^2 \frac{\partial \bar{T}(r_1)}{\partial r} \right]. \tag{2.25}$$

Eq. (2.25) shows that the loss of the superadiabatic heat flux between ICB and CMB equals the adiabatic cooling of the whole core

$$Q_a = \mathcal{H}(r_2) - \mathcal{H}(r_1) = 4\pi k \left[r_2^2 \frac{\partial \overline{T(r_2)}}{\partial r} - r_1^2 \frac{\partial \overline{T(r_1)}}{\partial r} \right] = -4\pi k \frac{\bar{T}_o}{r_o} \frac{d}{r_o} D_o \left[r_2^2 + r_1 r_2 + r_1^2 \right]. \tag{2.26}$$

Here we take (2.8) and (2.9) into account.

The value of the superadiabatic temperature is small. The superadiabatic heat flux on ICB can be increased if we assume that a thin temperature layer arises there with thickness

$$l \sim \frac{r_o}{\sqrt{Pe}}, \text{ where } \mathrm{Pe} = \frac{V_1 r_o}{\kappa_T}. \tag{2.28}$$

Then the jump of the superadiabatic temperature across the layer can be estimated from (2.25) as

$$\vartheta_1 = l\frac{Q_{sa}(r_1)}{4\pi r_1^2 k_T} = r_o Pe^{-1/2}\frac{Q_{sa}(r_1)}{4\pi r_1^2 k_T}. \tag{2.29}$$

We assume that this jump is of order of the typical value of ϑ. Estimation for the Archimedean flow velocity can be obtained directly from (2.10): $V_A = (\overline{g_o}/2\Omega)\alpha\vartheta_1$. We assume further that this velocity is of order of the typical flow velocity V_1. Then we obtain an estimate for ϑ_1:

$$\vartheta_1 = l\frac{2\Omega V_1}{\overline{g}_o\alpha} = Pe\frac{2\Omega\kappa_T}{D_o c_p}. \tag{2.30}$$

Equalizing two estimates (2.29) and (2.30) for ϑ_1 we obtain an equation for Pe. Its solution

$$Pe = \left[\frac{\kappa}{\kappa_T}\right]^{4/3}\left[\frac{r_o D_o Q_{sa}(r_1)}{8\pi\Omega r_1^2\kappa^2\overline{\rho}_o}\right]^{2/3}$$

can also be presented in the form:

$$Pe = Pe_c\hat{Q}_{sa}^{2/3}, \quad \text{where} \quad \mathrm{Pe}_c = \left[\frac{\kappa}{\kappa_T}\right]^{4/3} Ra^{2/3}, \qquad \hat{Q}_{sa} = \frac{Q_{sa}(r_1)}{Q_a}, \text{ and} \tag{2.31}$$

$$Ra = \frac{c_p D_o^2 \overline{T}_o}{2\Omega\kappa}R = \frac{\overline{g}_o\alpha\overline{T}_o r_o}{2\Omega\kappa}D_o R = \frac{\overline{g}_o\alpha\Delta\overline{T} r_o}{2\Omega\kappa}R = 4.8\times 10^{15} \tag{2.32}$$

is the well known Rayleigh number. Parameter R is defined by the expression

$$R = \frac{d}{r_o}\left[\frac{r_2^2}{r_1^2} + \frac{r_2}{r_1} + 1\right] = 11.5.$$

At the threshold of convection in the whole liquid core the superadiabatic heat flux on CMB is absent. So it follows from (2.25) that $Q_{sa}(r_1) = Q_a$. Respectively Pe_c is the critical value of Pe for the case when convection is situated in the whole core.

Now we can estimate the values of $\vartheta_1 = Pe(2\Omega\kappa_T / D_o c_p)$:

$$\theta_1 = \frac{\overline{T}_o}{Ra^{1/3}}D_o R\left[\frac{\kappa}{\kappa_T}\right]^{1/3}\hat{Q}_{sa}^{2/3} \tag{2.33}$$

and $V_1 = (\kappa_T / r_o)Pe$

$$V_1 = \frac{\kappa}{r_o} Ra^{2/3} \left[\frac{\kappa}{\kappa_T} \right]^{1/3} \hat{Q}_{sa}^{2/3}. \tag{2.34}$$

In fact, the momentum equation (2.10) presents the balance of the pressure, the Archimedean and Lorentz forces. The number of degrees of freedom of the pressure force is smaller than of the other two. That is why we assume that in some locations at least, this balance reduces only to the balance of the Archimedean and Lorentz forces

$$\bar{\rho}_o \bar{g}(r)\alpha\vartheta \sim \frac{\nabla \times \mathbf{B} \times \mathbf{B}}{\mu_o} \sim \frac{B^2}{\mu_o l_b}. \tag{2.35}$$

Further we will assume that the typical space scale of the magnetic field distribution l_b is the diffusive scale l defined by (2.28). Then this balance allows to estimate the typical value of the magnetic intensity and the electrical current J_1:

$$B_1 = \sqrt{2\Omega\bar{\rho}_o \kappa \mu_o} \left[\frac{\kappa}{\kappa_T} \right]^{1/3} Ra^{1/6} \hat{Q}_{sa}^{1/6}, \tag{2.36}$$

$$J_1 = \sqrt{\frac{2\Omega\bar{\rho}_o \kappa}{\mu_o r_o^2}} \left[\frac{\kappa}{\kappa_T} \right]^{1/2} Ra^{1/2} \hat{Q}_{sa}^{1/3}. \tag{2.37}$$

It is B_1 appropriate to compare our (IA) amplitudes defined by (2.33,34-36,37) with the results of computer simulations. Such comparison with the results of the anelastic computer simulation of Glatzmaier and Roberts (1996) (GR96) are given in Table 3.

Table 3

		V	**B**	**J**	l
IA	$12\times10^{-4}K$	$8.3\times10^{-4}ms^{-1}$	$11mT$	$0.12Am^{-2}$	$7.5\times10^{4}m$
GR96	$4\times10^{-4}K$	$5\times10^{-4}ms^{-1}$	$5mT$	-	-

Comparison betwe ϑ en the typical values of Glatzmaier and Roberts (1996) simulation and the amplitudes given by (2.33,34) and (2.36-37)

The difference of the specific entropy between ICB and CMB in GR96 computer simulation is of order of $\Delta S = 2\times10^{-4} Jkg^{-1}K^{-1}$. The correspondent value of the super-adiabatic temperature drop can be estimated by $\Delta\vartheta = \Delta S(\bar{T}_o / c_p) = 11\times10^{-4} Jkg^{-1}K$ which is close to our temperature amplitude, but the typical temperature of GR96 is two to three times smaller. The maximal values of the flow velocity and the intensity of magnetic field in this work are $20\times10^{-4}m/s$ and $20\times10^{-4}mT$, respectively. They approximately exceed two times our correspondent amplitudes, but the typical values of GR96 appear to be smaller than ours.

Though our super-adiabatic temperature unit (2.33) seems to be slightly overestimated, we conclude that our estimation is in a good qualitative agreement with the results of Glatzmaier and Roberts (1996).

As another test we use the Boussinesq computer simulation of Glatzmaier and

Roberts (1995). In comparison with their anelastic approximation the authors use a larger heat flux (almost of an order of magnitude larger than in GR96). At the same time the turbulent thermal diffusivity κ_T in this work is approximately with one order of magnitude smaller than in GR96. Results of their computer simulation in comparison with our amplitudes (2.33-37) are given in Table 4.

Table 4

	ϑ	V	**B**	**J**	l
IA	$1.1\times10^{-2}K$	$7.8\times10^{-3}ms^{-1}$	$12\,mT$	$0.7Am^{-2}$	$9.5\times10^{3}m$
GR95	$1.\times10^{-2}K$	$3\times10^{-3}ms^{-1}$	$15\,mT$	-	-

Comparison between the typical values of Glatzmaier and Roberts (1995) simulation and the amplitudes given by (2.33,34) and (2.36-37)

Table 4 shows that the amplitudes of ϑ and $\mathbf{V}$ increase with approximately one order of magnitude in comparison with these given by Table 2. Though our value of the flow velocity is at least two times larger than that of GR95, we believe that (2.33-37) are in qualitative agreement with the results of GR95.

We will use amplitudes (2.33-37) in order to non-dimensionalise the equations and boundary conditions of the problem. To do this, let us divide (2.10) by $2\Omega V_1$, (2.12) by ϑ_1/t_1, (2.4) by B_1/t_1 and (2.6) by $\vartheta_1/l = \vartheta_1/(r_o Pe^{-1/2})$, where the correspondent units are defined by (2.33-2.37). (As before, r_o and r_o/V_1 are used as the space and the time scales). Then we obtain:

$$R_o\left[\frac{\partial\widehat{\mathbf{V}}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}\right]=-\nabla\widehat{P}-\mathbf{1}_z\times\widehat{\mathbf{V}}+\mathbf{1}_r\hat{r}\hat{\vartheta}+\Lambda\hat{\mathbf{J}}\times\widehat{\mathbf{B}}+E\nabla^2\widehat{\mathbf{V}},\quad \nabla\cdot\widehat{\mathbf{V}}=0\,\mathbf{,} \qquad \text{(2.38,39)}$$

$$\frac{\partial\hat{\vartheta}}{\partial t}+(\widehat{\mathbf{V}}\cdot\nabla)\hat{\vartheta}=Pe^{-1}\nabla^2\hat{\vartheta}-Q_T+Q_J\hat{\mathbf{j}}^2-\hat{r}D_o\hat{\vartheta}\widehat{V}_r, \qquad \text{(2.40)}$$

$$\frac{\partial\widehat{\mathbf{B}}}{\partial t}=\nabla\times[\widehat{\mathbf{V}}\times\widehat{\mathbf{B}}]+\eta\nabla^2\widehat{\mathbf{B}},\quad \nabla\cdot\widehat{\mathbf{B}}=0, \qquad \text{(2.41,42)}$$

$$\frac{\partial\hat{\vartheta}}{\partial\hat{r}}=\sqrt{Pe}\hat{Q}_{sa}^{1/3},\qquad \hat{\vartheta}(\mathrm{r}_2)=0, \qquad \text{(2.43)}$$

where

$$R_o=Pe_c\mathrm{E}\frac{\kappa_T}{\nu_T},\qquad Pe=\left[\frac{\kappa_T}{\nu_T}\right]^{4/3}Ra^{2/3},\qquad E=\frac{\nu_T}{2\Omega r_o^2},\qquad \Lambda=\hat{Q}_{sa}^{-1/6},$$

$$Q_T=\frac{3\hat{Q}_{sa}^{-4/3}}{R\sqrt{Pe}},\qquad Q_j=\frac{\eta}{\kappa_T}\frac{D_o\hat{Q}_{sa}^{-2/3}}{\sqrt{Pe}},\qquad R_m=Pe_c\frac{\kappa_T}{\eta}\hat{Q}_{sa}^{2/3}.$$

If parameter $\hat{Q}_{sa}$ is smaller than 1, then the convection in the whole core is impossible, since in this case the super-adiabatic heat flux is too small to support the reference state. Fortunately, the whole heat flux on ICB exceeds the adiabatic heat flux there only with approximately 50%. Respectively $\hat{Q}_{sa} \sim 1.5$ for the Earth's core.

We choose the space density of the internal energy $\varepsilon^{\vartheta} = c_p \bar{\rho}_o \vartheta$ as a unit of space density for all types of energies (ε^k and ε^m). Then dividing (2.14-16) by $(V_1/t_1)\varepsilon_1^{\vartheta}$ we obtain the dimensionless equations

$$D_o \frac{R_o}{2}\frac{\partial \hat{\varepsilon}^k}{\partial t} = D_o \nabla \cdot \left[\underline{P\hat{\mathbf{V}}} + \frac{R_o}{2}\hat{\mathbf{V}}\hat{\varepsilon}^k - E\hat{\mathbf{V}} \times [\nabla \times \hat{\mathbf{V}}] \right] + D_o \left[\underline{\hat{r}\hat{V}_r\hat{\vartheta} + \hat{Q}_{sa}^{-1/6}\mathbf{V}\cdot[\mathbf{j}\times\mathbf{B}]} \right] - D_o E \left[\nabla \times \hat{\mathbf{V}} \right]^2 \tag{2.44}$$

$$\frac{\partial \hat{\varepsilon}^{\vartheta}}{\partial t} = \frac{D_o}{\sqrt{Pe}} \nabla \cdot \left[\sqrt{Pe}\hat{\mathbf{V}}\hat{\varepsilon}^{\vartheta} - \frac{\nabla \vartheta}{\sqrt{Pe}} + \frac{\hat{\mathbf{r}}}{\hat{Q}_{sa}R} \right] + \frac{D_o}{\sqrt{Pe}}\frac{\eta}{\kappa_T}\hat{\mathbf{j}}^2 + D_o E\left[\nabla \times \hat{\mathbf{V}}\right]^2 - D_o \hat{r}\hat{V}_r\hat{\vartheta} \tag{2.45}$$

$$\frac{D_o}{2\sqrt{Pe}}\frac{\partial \hat{\varepsilon}^m}{\partial t} = \frac{D_o}{\sqrt{Pe}} \nabla \cdot \left[[\mathbf{B}\times\mathbf{V}\times\mathbf{B}] + \frac{\eta}{\kappa_T}\frac{\hat{\mathbf{j}}\times\hat{\mathbf{B}}}{\sqrt{Pe}} \right] - D_o \left[\frac{\mathbf{V}\cdot[\mathbf{j}\times\mathbf{B}]}{\hat{Q}_{sa}^{1/6}} + \frac{\eta}{\kappa_T}\frac{\hat{\mathbf{j}}^2}{\sqrt{Pe}} \right] \tag{2.46}$$

Archimedean force converts heat into mechanical work. The efficiency of this process is restricted by the efficiency of Carnot circle. Taking into consideration the equaity

$$D_o \approx \frac{\Delta \bar{T}_o}{\bar{T}_o} \tag{2.47}$$

which follows from (1.6), we see that all terms in the equations for the kinetic and the magnetic energies are multiplied by the Carnot efficiency. Thus **the Carnot constraint is automatically incorporated into our energetic balance.** The amplitudes of the dimensionless energies are of order of 1. So the coefficients in front of them give their relative values. The internal energy ~ 1, the kinetic and the magnetic energies are respectively $\sim D_o(R_o/2) \sim 4\times 10^{-7}$ and $\sim (D_o/2)\sqrt{Pe} \sim 5.6\times 10^{-3}$.

All the terms in (2.44), except the underlined, are small. It follows from here that the work of the Lorentz force is of order of the work of the Archimedean one ($\sim D_o \sim 1/3$). The Ohmic dissipation in (2.46) seems to be much smaller ($\sim Pe^{-1/2}$) than the Lorentz force work. This probably means that the field is parallel to the flow.

Summering (2.44-46) we obtain the equation for the whole energy

$$\frac{\partial}{\partial t}\left[\varepsilon^{\vartheta} + \frac{D_o R_o}{2}\varepsilon^k + \frac{D_o}{\sqrt{Pe}}\varepsilon^m \right] = \nabla \cdot \mathbf{I}, \tag{2.48}$$

where

$$\mathbf{I} = \hat{\mathbf{V}}\varepsilon^{\vartheta} - Pe^{-1}\nabla\hat{\vartheta} - \frac{\hat{\mathbf{r}}}{R\sqrt{Pe}} - D_o \left[\frac{[\hat{\mathbf{B}}\times\hat{\mathbf{V}}\times\hat{\mathbf{B}}]}{\sqrt{Pe}} - \frac{\eta}{\kappa_T}\frac{\hat{\mathbf{j}}\times\hat{\mathbf{B}}}{Pe} - \hat{P}\hat{\mathbf{V}} - \frac{R_o}{2}\hat{\mathbf{V}}\hat{\varepsilon}^k - E\hat{\mathbf{V}}\times\left[\nabla\times\hat{\mathbf{V}}\right] \right] \tag{2.49}$$

is the energy flux**.**

The spectral (in κ_T) distributions (2.33-37) pretend to estimate the amplitudes of solution for all scales (the small κ_T means the small grid cell size in the computer simulation). If that is true, then the time derivatives of $\widehat{\mathbf{V}}$, $\hat{\vartheta}$ and $\widehat{\mathbf{B}}$ have to be independent of κ_T. In the opposite case, (2.33-37) would be destroyed by the time evolution even if the initial amplitude distributions satisfy it.

The turbulent diffusion term in the equation (2.40) and (2.45) depends on Pe and so on κ_T as well. To avoid these dependence we assume that turbulent diffusion is defined not by the large scale r_o, but by the small space scale $l = r_o/\sqrt{Pe_c}$. Then the first two terms in rhs of (2.40) are of the same order: $\nabla \cdot \widehat{\mathbf{V}}\hat{\vartheta} \sim Pe^{-1/2}\nabla \cdot Pe^{-1/2}\nabla \cdot \hat{\vartheta}$.

In this connection it is appropriate to cite Glatzmaier and Roberts (1996) who, describing their computer simulation, write: *Diffusive heat flux due to unresolved turbulence* [$\mathbf{q}_\vartheta = Pe_c^{-1/2}\nabla\hat{\vartheta}$] *is driven by the entropy* [*the super-adiabatic temperature*] *gradient; whereas the conductive heat flux down adiabatic* [$\mathbf{q}_T = \kappa\nabla\overline{T}(r) \to (\hat{\mathbf{r}}/R)Pe_c^{-1/2}$] *is driven by the reference state temperature gradient. Although these two heat fluxes are comparable* [$|\mathbf{q}_\vartheta| \sim (1/2)Pe_c^{-1/2}$ *and* $\mathbf{q}_T \sim 10^{-1}Pe_c^{-1/2}$] *in our simulation, the divergence of the turbulent heat flux* [$\nabla \cdot \mathbf{q}_{\hat{\vartheta}} = Pe_c^{-1}\nabla^2\hat{\vartheta} \sim 1/2$] *is almost two orders of magnitude greater than the divergence of conductive heat flux* [$\nabla \cdot \mathbf{q}_{\hat{T}} = (3/R)Pe_c^{-1/2} \sim 8.6\times10^{-3}$]. *The divergence of the convective heat flux* [$\nabla \cdot \widehat{\mathbf{V}}\hat{\vartheta} \sim 1/2$] *(due to the resolved large scale) is comparable to the divergence of the turbulent heat flux* [$\sim Pe_c^{-1}\nabla^2\hat{\vartheta} \sim 1/2$] *(due to parameterized unresolved small scales); they typically have opposite signs at a given location and are largest near the boundaries*.

Here in the square brackets we give our correspondent values. Since our super-adiabatic temperature unit is rather overestimated, we adopt for estimations $\hat{\vartheta} = 1/2$. Why are $\mathbf{q}_{\hat{\vartheta}}(\hat{\mathbf{r}})$ and $\mathbf{q}_{\hat{T}}(\hat{r})$ "comparable"? The whole heat flux (averaged over long enough time periods) conserves. This flux on ICB is mainly a super-adiabatic one, but on CMB it is predominantly adiabatic. In the core, the super-adiabatic heat flux gradually converts into adiabatic and so they are of the same order. (This is just the reason why we cannot neglect this term in (2.40) in spite of its relative smallness.) Why do these "comparable" fluxes have different divergences? The reason is the small space scale of the super-adiabatic temperature.

Thus we conclude that the terms in our heat transport equation (2.40) are in a qualitative agreement with Glatzmaier and Roberts's (1996) computer simulation.

In neglecting the small, $O(E)$ and $O(R_o)$, terms in (2.38) we obtain that $\widehat{\mathbf{V}}$ can be expressed in terms of the Archimedean $\sim\hat{\vartheta}$ and the magnetic $\sim\left[\hat{\mathbf{j}}\times\widehat{\mathbf{B}}\right]$ velocities. The first of them does not depend on κ_T, but the second one can depend on it through the Roberts number q in (2.41), providing that the small space scale for the magnetic field distribution is the same $l = r_o/\sqrt{Pe_c}$ as that for the temperature. Therefore, we cannot answer the question if the flow velocity (2.34) and the magnetic field (2.36,37) spectra are correct for $\kappa_T \ll \eta$.

The units of $\mathbf{V}$, $\mathbf{B}$ and ϑ in (2.38-43) are chosen in a way to make the dimensionless amplitudes of these variables of order of 1. This choice also assumes that the time scale is defined by the flow velocity: $t_1 = L_1 / V_1 = r_o / V_1$. However, most of the colleagues use the diffusional time scale for which $t_1 = L_1^2/\kappa_T = r_o^2/\kappa_T$. Then the unit of the flow velocity turns into κ_T / L_1 and respectively its amplitude enhances: $\widehat{V} \sim A$ where $A = Pe_c \hat{Q}_{sa}^{2/3}$ and $Pe_c = 975$, but the amplitudes of $\hat{\vartheta}$ and $\widehat{B}$ keep their values ~ 1. To transform equations (2.38,40,41) to this type of scaling, we have to multiply them by Pe . Then we obtain:

$$\frac{E}{\Pr}\left[\frac{\partial \widehat{\mathbf{V}}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right] = -\nabla\widehat{P} - \mathbf{1}_z \times \widehat{\mathbf{V}} + \mathbf{1}_r A\hat{r}\hat{\vartheta} + A\hat{Q}_{sa}^{-1/6}\hat{\mathbf{J}} \times \widehat{\mathbf{B}} + E\nabla^2\widehat{\mathbf{V}}, \qquad (2.50)$$

$$\frac{\partial\hat{\vartheta}}{\partial t} + (\widehat{\mathbf{V}}\cdot\nabla)\hat{\vartheta} = \nabla^2\hat{\vartheta} - Q_T + Q_J\,\mathbf{j}^2 - \hat{r}D_o\hat{\vartheta}\widehat{V}_r, \qquad (2.52)$$

$$\frac{\partial\widehat{\mathbf{B}}}{\partial t} = \nabla\times[\widehat{\mathbf{V}}\times\widehat{\mathbf{B}}] + q^{-1}\nabla^2\widehat{\mathbf{B}}, \qquad \nabla\cdot\widehat{\mathbf{B}} = 0, \qquad (2.53,54)$$

$$\frac{\partial\hat{\vartheta}}{\partial\hat{r}} = \sqrt{Pe_c}\,\hat{Q}_{sa}^{1/3}, \qquad \hat{\vartheta}(\mathrm{r}_2)=0\,, \qquad (2.43)$$

where

$$A = Pe_c\hat{Q}_{sa}^{2/3} = 975\hat{Q}_{\mathrm{sa}}^{2/3}, \qquad \mathrm{Pe}_c = \left[\frac{\kappa}{\kappa_T}\right]^{4/3} Ra^{2/3} = 975, \qquad \mathrm{E} = \frac{\nu_T}{2\Omega r_o^2} = 2.6\times10^{-9}$$

$$Q_T = \frac{3}{R}\frac{\sqrt{Pe_c}}{\hat{Q}_{sa}} = \frac{8.1}{\hat{Q}_{sa}}, \qquad \mathrm{Q}_j = \frac{D_o}{q}\sqrt{Pe_c}\,\hat{Q}_{sa}^{-1/3} = q^{-1}\frac{8.75}{\hat{Q}_{sa}^{1/3}}, \qquad \mathrm{q} = \frac{\kappa_T}{\eta}, \qquad \Pr = \frac{\nu_T}{\kappa_T}$$

Note that the number of degrees of freedom in these equations is very small. In fact, only two parameters, $\hat{Q}_{sa}$ and q, can be variable. In the estimate above we choose $\kappa_T = \nu_T = \eta = 2m^2s^{-1}$.

Coefficient A at the super-adiabatic temperature in (2.50) is commonly called the Raleigh number Ra. Its value is widely discussed in literature (see e.g. Jones (2000) or Gubbins (2001)). In our estimate A equals Pe and is expressed through Ra. However, it is much smaller than Ra. That is why we use another name for this value, the Archimedean number.

Conclusion

Relatively powerful computers appeared in the last quarter of the former century, which allowed to start computer simulations of the geodynamo. Most of these simulations were carried out in the frame of the incompressible Boussinesq approximation. The reason for this is the relatively small ($\sim$ 20%) compressibility of the Earth's core.

The essential difference between the Earth's core and the laboratory convections can be explained in the following manner. The compressibility of the convective layer is defined (see e.g. Braginsky and Roberts (1995)) by parameter $D = \overline{g}_o \alpha d / c_p$ \$, where d is the thickness of the layer. This parameter is small for the laboratory convection due to the small values of d. However, for the Earth's core D is big enough (~ 0.27), since d is very large.

In the present work we show that the Boussinesq approximation is an incorrect model for the Earth's core convection, because its heat transport equation neglects the essential part of the heat flux, the adiabatic one. It has been shown in section 1 that this heat flux does not vanish in the incompressible limit, $D \to 0$. It creates an additional cooling in the equation for the superadiabatic temperature.

The work of the Archimedean force in the Boussinesq approximation is assumed to be negligibly small. Therefore, the Boussinesq's energy conservation law takes the form of heat conservation only. Here we found out that the efficiency of the convection, when converting the heat into energy of the magnetic field, is of order of the efficiency of Carnot circle. This efficiency for the Earth's core is not small $\sim D$ and so it has to be taken into account. The work of the Archimedean force also forms an additional cooling source in the equation for the superadiabatic temperature.

In the present work we propose a new Incompressible Approach which takes into consideration both of these effects (the adiabatic and the Archimedean cooling). Since the work of the Archimedean force is not neglected in our method, the heating, which is due to the Ohmic dissipations, is not small and is included in the heat transport equation as well. (We assume that this term can enhance essentially the dynamic of the flow). As a result, we obtain the law of the energy conservation (2.48), which takes into account all forms of energy, not only the heat. This equation in its integral form can be used for the control of the solution during the numerical simulations.

The Boussinesq approximation does not have its own unit for the temperature. That is why the temperature difference $\Delta\overline{T}$ between ICB and CMB is used for temperature scaling. As a result, an enormous Rayleigh number in the momentum equation arises as a coefficient at the Archimedean force. Its unrealistic value is widely discussed in literature (see e.g. Jones (2000) or Gubbins (2001)).

By using the non-linearity of the heat transport equation we were able to estimate the amplitudes of the unknown variables (the flow velocity, the adiabatic temperature etc). These amplitudes have been used for non-dimensionlazation of the problem. As a result, our coefficient at the Archimedean force takes a moderate value of order of the Peclet number Pe, instead of the enormous Rayleigh number in the Boussinesq approximation. For this value we use another name, the Archimedean number.

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Приближение на несвиваем флуид към конвекцията в земното ядро

А. П. Ануфриев

Резюме. Магнитното поле на Земята се създава от конвективни течения в нейното електропроводящо течно ядро. Съответно уравненията за термичната конвекция са съществена част от проблема за генерацията на това поле. Тъй като свиваемостта на ядрото е малка, конвекцията в него обикновено се изучава в приближението на Бус-синеск (БП), пренебрегващо свиваемостта на течността. Този подход, обаче, неявно се пренебрегва големият кондуктивен топлинен поток свързан с разпределение на температура в адиабатичното референтно състояние на ядрото. В настоящата работа се предлага нов несвиваем подход, който отчита влияние на този ефект в уравнението за топлинен пренос. В това уравнение възникват два нови члена, наречени от нас адиабатично и Архимедово охлаждане. Първият от тях описва топлина, поддържаща адиабатичен профил на температурно разпределение на адиабатичното референтно състояние. Вторият член е свързан с тази част от топлината, която се превръща в механична работа, създаваща магнитно поле. Всичките тези нови членове зависят от разликата на плътността между горната и долната граници на ядрото и изчезват, ко-гато тази разлика клони към нула.