ADIABATIC APPROXIMATION IN THE GEODYNAMO CONVECTION

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Abstract. Magnetic field of the Earth is created in the its conductive liquid core by convective flows. Respectively equations of the thermoconvection are the important part of the generation problem. Commonly convection in the Earth core is studied in the Boussinesq approximation neglecting the compressibility of the liquid. However, compressibility plays the crucial role in this convection and especially in the energy balance. Without compressibility this balance reduces to the balance of the heat only. That is why the generalization of BA towards the compressible case seems to be important. We present here new equations for the compressible convection. The main diffe3rnce from the Boussinesq equations are as follows. In the heat transport equation two new terms come into existence, the adiabatic and the Archimedean cooling. The first of them describes heat supporting the adiabatic temperature distribution of the reference state. The second one is connected with that part of the heat which is converted into the mechanic work creating the magnetic field. One new term arises in the momentum equation as well. This term describes the additional buoyancy due to compressibility of the liquid. All the new terms depend on the density difference of liquid between the bottom and upper boundaries of the liquid core. They vanish when this difference tends to zero. We believe that our equations have to be especially essential for stars where this difference in their convective zones is enormous.

Key words: anelastic convection, Boussinesq approximation, dynamo theory, compressibility.

Introduction

Convection plays a very important role in the life of cosmic bodies such as planets and stars. Very often it is studied in the Boussinesq approximation (BA) where compressibility is neglected. However compressibility is essential even for the liquid bodies such as the mantle or the liquid core of the Earth. That is why the generalization of BA towards the compressible case seems to be important.

The equations of BA for the buoyancy force directed along the polar radius can be written in a form:

$$\rho \frac{d\mathrm{V}}{dt} = -\nabla p + 1_r\, g\rho\alpha T, -2\rho\Omega \times \mathrm{V} + \mathrm{F}^b + \mathrm{F}^\upsilon \qquad \nabla \cdot \mathrm{V} = 0, \qquad (1.1,\ 2)$$

$$\frac{\partial T}{\partial t} + (\mathrm{V}\cdot\nabla)T = \frac{1}{C_p\rho}\nabla\cdot k\nabla T + \frac{Q}{C_p\rho} \qquad (1.3)$$

Here **V** is the flow velocity, ρ is density, *p* is pressure, *g* is the gravitational acceleration, is the coefficient of thermal expansion, *T* is temperature, C_p is specific heat at constant pressure, k is thermal conductivity, *Q* is the rate of heat addition per unit volume by internal heat sources. The Lorentz force is defined by the expressions: $\mathrm{F}^b = \mathrm{J}\times\mathrm{B}, \mathrm{J} = \nabla\times\mathrm{B}/\mu_o$. The viscous force is defined by the tensor of viscous tension: $\mathrm{F}^\upsilon = \nabla\cdot\sigma'$.

It is crucial for BA that the density variations are small and depend on the temperature only and not on the pressure. Accordingly the buoyancy force depends only on the temperature.

However the densities and temperatures into stars and planets depend essentially on position. How does the BA simple picture change if the compressibility is taken into account?

Let us represent the quantities in the body as a sum of their Reference State values and their disturbances due to convection:

$$\rho = \overline{\rho}(r) + \rho'(\mathrm{r},t), \qquad T = \overline{T}(r) + \vartheta(\mathrm{r},t), \qquad P = \overline{P}(r) + p'(\mathrm{r},t),$$
$$\Psi = \overline{\Psi}(r) + \psi(\mathrm{r},t), \qquad (1.4)$$

where ψ is the gravitational potential.

If following a naive point of view we should replace the temperature in BA equation (1.1) with its expression from (1.4) and then obtain the Arhimedean force in a form: $1_r\, g\rho\alpha\overline{T} + 1_r\, g\rho\alpha\vartheta.$ In the same case the heat transport equation (1.3) would take a form:

$$\frac{\partial\vartheta}{\partial t} + (\mathrm{V}\cdot\nabla)\vartheta + (\mathrm{V}\cdot\nabla)\overline{T}(r) = \frac{1}{C_p\overline{\rho}}\nabla\cdot k\nabla\vartheta + \frac{1}{C_p\overline{\rho}}\nabla\cdot k\nabla\overline{T}(r) + \frac{Q}{C_p\overline{\rho}} \qquad (1.5)$$

Here the term $(\mathrm{V}\cdot\nabla)\overline{T}(r)$ appears. It is often used in the Boussinesq approximation and is called the β-term. Does this term have a physical meaning? To check this let us carry out a thought experiment. Let us imagine a small liquid element emerging in the liquid. Ascending it carries out work and it cools due to this work. So for existing of the convection it is not enough that the temperature of the element is higher than the temperature of the liquid above it. The element would continue to emerge only if its temperature after this cooling is not lower than the temperature of its new environment.

If neglecting the heat diffusion from or into the element its temperature would change adiabatically. That is the reason for the temperature profile for zero buoyancy force to be called adiabatic. In the adiabatic profile the ascending element changes its temperature fitting it to the adiabatic temperature of its new environment. That is why adiabatic temperature is not transferred by the flow. Only if the element's temperature is higher than the adiabatic temperature, a buoyancy force arises acting on it. And this force would be proportional to the superadiabatic temperature and not to the whole temperature as it follows from the "naive" point of view.

Thus the same reason, which makes the buoyancy force proportional to the superadiabatic temperature, leads to lack of the transport of the adiabatic temperature due to the flow velocity. Nevertheless everybody usually writes correctly the buoyancy force which is proportional to the superadiabatic temperature, but at the same time many authors include the transport of the adiabatic temperature in their heat transport equation.

Another new term included in (1.5) is $\nabla \cdot k\nabla\overline{T}(r)$ For superadiabatic temperature this term plays the role of an additional heat source. What is the physical meaning of this term and must it really be included in the equation? Let us multiply (1.5) by $C_p\overline{\rho}$ and integrate it over the whole convective shell. Let us neglect the rate of the heat addition Q. Then we obtain the internal energy balance in a form:

$$\frac{\partial E}{\partial t} = 4\pi r_1^2\left(I^{sa}(r_1) + I^a(r_1)\right) - 4\pi r_2^2\left(I^{sa}(r_2) + I^a(r_2)\right) \tag{1.6}$$

where E is the internal energy and I^a, I^{sa} are surface densities of the adiabatic and superadiabatic diffusion heat fluxes:

$$E = \iiint C_p\overline{\rho}\vartheta d^3r \qquad I^{sa}(r_i) = \frac{k}{4\pi r_i^2}\iint\frac{\partial\vartheta}{\partial n_i}dS_i \qquad I^a(r_i) = k\frac{\partial\overline{T}(r_i)}{\partial r} \tag{1.7}$$

Here indexes 1 and 2 correspond to the bottom and upper boundaries of the shell.
Under time average the lhs of (1.6) vanishes. Averaging (1.6) over time we get:

$$4\pi r_1^2 I^{sa}(r_1) - 4\pi r_2^2 I^{sa}(r_2) = 4\pi r_2^2 I^a(r_2) - 4\pi r_1^2 I^a(r_1) \sim k\frac{\Delta\overline{T}}{d}4\pi\left(r_2^2 - r_1^2\right) \tag{1.8}$$

where $\Delta\overline{T}$ is the adiabatic temperature difference between the upper and the bottom boundaries and d is the thickness of the shell.

The whole adiabatic heat flux on the upper boundary is larger than that on the bottom boundary at least because its surface is smaller. The difference between these is countervailed by the superadiabatic heat flux from the bottom boundary. That is why the superadiabatic heat flux decreases from the bottom to the upper boundary and the term $\left(1/C_p\overline{\rho}\right)\nabla \cdot k\nabla\overline{T}(r)$ in (1.5) describes the "cooling" of this heat flux.

In different papers the heat transport equation for compressible liquid is used in different forms. How is it possible to compare our results if even the equations we use are not unified? Above we have tried to estimate what the proper equation should be like. The goal of this paper is to derive the true equations for the compressible convection and to check our rather intuitive estimate.

Basic Equations and Reference State

We start from momentum and continuity equations

$$\rho \frac{d\mathrm{V}}{dt} = -\nabla P - \rho \nabla \Psi - 2\rho \Omega \times \mathrm{V} + \mathrm{F}^b + \mathrm{F}^\upsilon \tag{2.1}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\mathrm{V}\rho) \tag{2.2}$$

the equation of state and Newton equation:

$$\rho = \rho(P,T), \qquad \Delta\Psi = 4\pi G\rho \tag{2.3,4}$$

heat transport

$$\rho T \frac{dS}{dt} = C_p \rho \frac{dT}{dt} = \nabla \cdot (k \nabla T) + Q, \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathrm{V} \cdot \nabla \tag{2.5}$$

Maxwell's and Ohm's equations:

$$\frac{\partial \mathrm{B}}{\partial t} = \nabla \times \mathrm{E} \qquad \mathrm{E} = \mathrm{J}/\sigma - \mathrm{V} \times \mathrm{B} \tag{2.6,7}$$

Here **E** is the intensity of the electrical field and σ - is the electrical conductivity. The heat transport equation can be also written in a form (see G. Shubert et al (2001)):

$$C_\upsilon T \frac{d ln T \rho^{-\gamma}}{dt} = \nabla \cdot (k \nabla T) + Q, \tag{2.8}$$

where Gruneisen parameter , adiabatic compressibility and volume expansion α are defined by the formulas:

$$\gamma = \frac{\alpha}{\rho C_p \chi_a}, \qquad \chi = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_s, \qquad \alpha = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Time derivatives in the momentum and the heat transport equations are essential for very powerful processes such as the genesis of stars or explosion of new stars. Movements into steady planets and stars create only small disturbances of their steady parameters such as density, temperature, pressure and gravitational potential. That is why the whole solution can be divided in two parts: the steady Reference State (RS) and the convection. Further we will overbar the symbol referring to the quantities of the RS and will assume that convective quantities are small comparatively to the RS ones:

$$\frac{\vartheta}{\overline{\overline{T}}} << 1 \qquad \frac{p}{\overline{\overline{P}}} << 1 \qquad \frac{\rho}{\overline{\overline{\rho}}} << 1 \qquad \frac{\psi}{\overline{\overline{\Psi}}} << 1 \tag{2.9}$$

Convective values are defined by the convective heat flux: the more heat flux the more convective quantities. At which value of the heat flux would the relation (2.9) be violated? To answer this question we can use the estimate for the convective temperature ϑ and the flow velocity **V**:

$$\vartheta \sim \varepsilon^{\frac{2}{3}} \left(\frac{k}{k_T}\right)^{\frac{1}{3}} \left(\frac{r_2}{r_1}\right)^{\frac{4}{3}} \frac{\Delta \overline{T}}{Ra^{\frac{1}{3}}} \qquad V_1 \sim \varepsilon^{\frac{2}{3}} \left(\frac{k}{k_T}\right)^{\frac{1}{3}} \left(\frac{r_2}{r_1}\right)^{\frac{4}{3}} \frac{\alpha \Delta \overline{T}}{Ra^{\frac{1}{3}}} \frac{\overline{g}}{2\Omega}$$

$$Ra = \frac{\overline{g} \alpha d \Delta \overline{T}}{2\Omega k} \tag{2.10}$$

where r_1 and r_2 are the bottom and upper boundaries of the convective shell, $\Delta \overline{T}$ is the thickness of the shell, $d = r_2 - r_1$ is the temperature difference between the boundaries, k and k_T are the molecular and the turbulent diffusivities, $\overline{g}$ is the gravitational acceleration, Ω is the angular velocity of the Earth and ε is the relation of the whole heat flux to its adiabatic component. For the Earth's core $\varepsilon \sim 1$. To violate the condition (2.9) for temperature, ε must be at least of the order of $\sqrt{Ra}$ where $Ra \sim 10^{14}$ is the enormous Raylegh number. So for the Earth's core (and any planetary) convection relation (2.9) for the temperature is satisfied with large precision. Estimations for the pressure and the density give:

$$\frac{p}{\overline{\overline{P}}} \sim \frac{2\Omega V \overline{\rho} L}{g \overline{\overline{\rho}} L} = \frac{2\Omega V}{g} \sim \varepsilon^{\frac{2}{3}} \left(\frac{k}{k_T}\right)^{\frac{1}{3}} \left(\frac{r_2}{r_1}\right)^{\frac{4}{3}} \frac{\alpha \Delta \overline{T}}{Ra^{\frac{1}{3}}} \qquad \frac{\rho}{\overline{\overline{\rho}}} \sim\sim \frac{\alpha \vartheta \overline{\rho}}{\overline{\overline{\rho}}} = \frac{\vartheta}{\overline{\overline{T}}} \alpha \overline{T} \tag{2.11}$$

For the perfect gas $\alpha T{=}1$, but for liquid $\alpha T{<<}1$. Even if we neglect the smallness of this parameter as well the smallness of κ/κ_T, we again obtain incredible relation $\varepsilon \sim \sqrt{Ra}$ as a condition for violating of (2.9). Thus we can conclude that (2.9) is satisfied for convection with large precision. Why are the superadiabatic disturbances so small? The reason is in the convective heat transport. It is so effective that an enormous amount of heat can be transported without essential increasing of the temperature.

Let us discuss the RS. The momentum equation for RS reduces to the balance of the gravitational and the pressure forces. In neglecting of centrifugal force the quantities of the RS must be spherically symmetric, i.e. depending on r only. So equations (2.1,3,4) take a simple form:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \overline{\Psi}}{\partial r}\right) = 4\pi G \overline{\rho}, \qquad \frac{\partial \overline{P}}{\partial r} = -\overline{\rho} \frac{\partial \overline{\Psi}}{\partial r}, \qquad \rho(r) = \rho(P(r), T(r)) \tag{2.12-14}$$

These equations are steady and one can expect that RS is a steady solution. However, generally speaking, it is not true since the heat transport equation essentially depends on time. Simply speaking, planets are cooled. To have a steady solution this equation needs heat support. It can be an additional volume heating $\overline{Q}$ or a heat flux on the bottom. For simplicity let us assume that the additional heating sources in the convective shell are absent and the time independent heat flux I_1 is the only heat source in the (2.5). If this flux is smaller than the value of adiabatic heat flux at r_1 then (2.5) takes the form:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial \overline{T}}{\partial r}\right)=0 \qquad k\frac{\partial \overline{T}(r_1)}{\partial r}=-\frac{I_1}{4\pi r_1^2} \qquad \overline{T}(r_1)=\overline{T_1} \qquad (2.15\text{-}17)$$

It is easy to check that solution (2.15-17) has the form:

$$\overline{T}(r)=\overline{T}_1-\frac{I_1}{4\pi r_1 k}\frac{r-r_1}{r} \qquad \textit{and} \qquad \Delta\overline{T}=\overline{T}(r_1)-\overline{T}(r_2)=\frac{I_1}{4\pi r_1 k}\frac{d}{r_2} \qquad (2.18)$$

This solution is valid until the temperature gradient on the bottom boundary is smaller than the adiabatic one. In the opposite case $\left(I(r_1)>I^a(r_1)\right)$ convection arises. The main difference between compressible convection and incompressible Boussinesq one is the convective region. In the Boussinesq case convection arises in the whole convective shell. The region of compressible convection is smaller if the heat flux on the bottom boundary is smaller than the adiabatic heat flux on the upper boundary $\left(I(r_1)<I^a(r_2)\right)$. Indeed it is well known and it will be shown in the next section that adiabatic temperature is governed by the equation:

$$\frac{1}{\overline{T}^a}\frac{d\overline{T}^a}{dr}=-\frac{\alpha\overline{g}}{C_p} \qquad (2.19)$$

For the Earth's core the gradient of the adiabatic temperature is approximately constant and so the adiabatic heat flux $I^a(r)$ increases being proportional to $4\pi r^2$. If $I(r_1) < I^a(r_2)$, then some r^* exists in which $I^a(r^*) = I(I(r_1)$. In the region $r_1 < r < r^*$ the temperature is superadiabatic and convection exists. Region $r^* < r < r_2$ is with under adiabatic heat flux and temperature. Convection is impossible here. Thus generally speaking, two regions exist in the shell: the convective one in which RS is defined by equations (2.12-14,19) and the inconvective, governed by (2.12-14,18). Only if $I(r_1) > I^a(r_2)$ the whole shell is convective.

The presence of additional heat sources can change this situation. For example, if heat flux on the bottom boundary is smaller than the adiabatic one, convection near the boundary is absent. However additional heat sources can create heat flux which at any level r^* exceeds adiabatic heat flux. Then the region $r_1 < r < r^*$ would be inconvective and region $r^* < r < r_2$ would be convective. May be this picture is realized in the Mantle?

Convection equations

As it has been emphasized above, convection represents a such type of solution which can be decomposed into two parts: the stationary Reference State and the small convective quantities (1.4). Taking into account the appliance to stars and planets we demand additionally for RS to be radially dependent only. Substitution of (1.4) in (2.1) yields:

$$\bar{\rho}\frac{d\mathrm{V}}{dt} = -\nabla\left(\bar{P}+p\right)-\left(\bar{\rho}+\rho\right)\nabla\left(\bar{\Psi}+\psi\right)-2\bar{\rho}\Omega\times\mathrm{V}+\mathrm{F}^{b}+\mathrm{F}^{\upsilon} \tag{3.1}$$

First we extract from this equation large terms of RS. Eq. (3.1) for RS values is reduced to balance between the pressure and the gravitational forces (2.13). Small imbalance of these forces leads to convection:

$$-\nabla\left(\bar{P}+p\right)-\left(\bar{\rho}+\rho\right)\nabla\left(\bar{\Psi}+\psi\right)-\left(-\nabla\bar{P}-\bar{\rho}\nabla\bar{\Psi}\right)=-\nabla p-\bar{\rho}\nabla\psi-\rho\nabla\bar{\Psi} \tag{3.2}$$

The Convective quantity ρ consists of two parts: ρ^{ϑ} depending on superadiabatic temperature ϑ and ρ^{p} depending on convective pressure p :

$$\rho=\rho^{\vartheta}+\rho^{p}=\frac{\partial\bar{\rho}}{\partial\bar{T}}\vartheta+\frac{\partial\bar{\rho}}{\partial\bar{P}}p=-\alpha\bar{\rho}\vartheta+p\left(\frac{\partial\bar{\rho}}{\partial r}\right)\Big/\left(\frac{\partial\bar{P}}{\partial r}\right)=-\bar{\rho}\alpha\vartheta-\frac{p}{g\bar{\rho}}\frac{\partial\bar{\rho}}{\partial r}$$

Respectively

$$-\nabla p-\bar{\rho}\nabla\psi-1_{r}\bar{g}\left(\rho^{\vartheta}+\rho^{p}\right)=-\nabla p-\bar{\rho}\nabla\psi+1_{r}\overline{g\rho}\alpha\vartheta+\frac{p}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial r}=$$

$$-\bar{\rho}\nabla\frac{p^{*}}{\bar{\rho}}+1_{r}\overline{g\rho}\alpha\vartheta \qquad\qquad p^{*}=p-\bar{\rho}\psi$$

Finally we can write the momentum equation in a form close to that of Braginsky and Roberts (1995):

$$\bar{\rho}\frac{d\mathrm{V}}{dt}=-\bar{\rho}\nabla\frac{p^{*}}{\bar{\rho}}+1_{r}\overline{g\rho}\alpha\vartheta-2\bar{\rho}\Omega\times\mathrm{V}+\mathrm{F}^{b}+\mathrm{F}^{\upsilon} \tag{3.3}$$

The equation of continuity can be obtained from (1.2) if one takes into account that $\bar{\rho}$ does not depend on time:

$$\nabla\cdot\mathrm{V}\bar{\rho}=0 \tag{3.4}$$

Here we neglect $\partial\rho/\partial t\sim\rho/T$ in comparison with $\nabla\cdot\mathrm{V}\bar{\rho}\sim V\bar{\rho}/L$ where T, L and V are the typical scales of time, space and flow velocity and ρ is the typical value of the convective density. This neglecting is valid since it is equivalent to obvious inequality: $TV/L\sim 1>>\rho/\bar{\rho}$

To obtain the heat transport equation we substitute the same decomposition (1.4) into (2.5):

$$C_p \frac{d(\bar{T}+\vartheta)}{dt} - \frac{(\bar{T}+\vartheta)\alpha}{\bar{\rho}+\rho}\frac{d(P+p)}{dt} = \frac{1}{\bar{\rho}+\rho}\nabla\cdot k\nabla(\bar{T}+\vartheta) + Q \qquad (3.5)$$

The convective quantities as we have shown in Section 2 are much smaller than the adiabatic ones. So we can decompose (3.5) and neglect the small quadratic terms.

$$(\mathbf{V}\nabla)\bar{T} - \frac{\alpha\bar{T}}{C_p\bar{\rho}}(\mathbf{V}\nabla)\bar{P} =$$

$$-\frac{d\vartheta}{dt} + \frac{\alpha\bar{T}}{C_p\bar{\rho}}\left[\frac{dp}{dt} + \left(\frac{\vartheta}{\bar{T}} - \frac{\rho}{\bar{\rho}}\right)\frac{d\bar{P}}{dt}\right] + \frac{1}{C_p\bar{\rho}}\nabla\cdot k\nabla\bar{T} + \frac{1}{C_p\bar{\rho}}\nabla\cdot k\vartheta + \frac{Q}{C_p\bar{\rho}} \qquad (3.6)$$

The lhs of this equation represents in fact the time derivative of the entropy of small liquid element (see (2.5)). It is defined by small convective terms and by the heat diffusion from the element. The first can be obviously neglected in comparison with lhs. Diffusion can also be neglected if Peclet number, $Pe=VL/k>>1$. Here $k=k/C_p\rho$ is the thermal diffusivity and V and L are the typical space and velocity scales. This condition is always satisfied for adiabatic RS. Then in the first approximation we obtain:

$$\mathbf{V}\cdot\left[\nabla\bar{T} - \frac{\alpha\bar{T}}{\bar{\rho}C_p}\nabla\bar{P}\right] = C_p\bar{T}V_r\left[\frac{1}{\bar{T}}\frac{d\bar{T}}{dr} - \frac{\alpha}{\bar{\rho}C_p}\frac{d\bar{P}}{dr}\right] = C_p\bar{T}V_r\left[\frac{1}{\bar{T}}\frac{d\bar{T}}{dr} + \frac{\alpha\bar{g}}{C_p}\right] = 0$$

Two possibilities arise here. In the convective region $V_r \neq 0$. Then we obtain that the condition for the existence of convection is the adiabatic temperature distribution of the RS:

$$\frac{1}{\bar{T}}\frac{d\bar{T}}{dr} = -D \qquad\qquad D = \frac{d\alpha\bar{g}}{C_p} \qquad (3.7)$$

where d is the thickness of the shell and D plays the role of non-dimensional gravitational acceleration.

Another possibility can be realized if the temperature distribution does not satisfy (3.7) i.e. is under adiabatic. Then free convection is absent: $V_r=0$. It does not mean that the flow is absent in this region. It means that only two-dimensional flows are allowed here. A liquid element cannot ascend or descend in the under-adiabatic region (if it exists). It means that Arhimedean force does not include energy into momentum equation.
So in this region energy is only dissipated. This effect has no analogy in incompressible convection.

Neglecting lhs in (2.8) we obtain adiabatic equation connecting the temperature with the density:

$$\frac{\overline{T}(r)}{\overline{T}_1} = \left(\frac{\overline{\rho}(r)}{\overline{\rho}_1}\right)^{\gamma}$$

From here it follows that

$$\frac{1}{\overline{\rho}}\frac{d\overline{\rho}}{dr} = \frac{1}{\gamma T}\frac{d\overline{T}}{dr} = -\frac{D}{d\gamma} \tag{3.8}$$

Equations (3.7) and (3.8) allow to estimate drops of the RS temperature and density over the shell:

$$\Delta\overline{T} \sim \overline{T}D \qquad\qquad \Delta\overline{\rho} \sim \overline{\rho}\frac{D}{\gamma} \tag{3.9}$$

It follows from (3.9) that D is the measure of the compressibility in the problem. The incompressible limit corresponds to $D \to 0$. We accept the following value of parameters for the liquid Earth core: $\alpha = 10^{-5} K^{-1}$, $d = 2.26 \times 10^6 m$,$C_p = 840 Jkg^{-1} K^{-1}$. The preliminary reference Earth model (PREM) of Dziewonski and Anderson (1981) gets values for $g(r)$ changing from $4.4 ms^{-2}$ on the inner core boundary (ICB) to $10.7 ms^{-2}$ on the core mantle boundary (CMB). So $D(r)$ changes from 0.11 to 0.28. Following Glatzmaier and Roberts (1996) we accept $\gamma = 1.35$.
Taking (3.7) into account we obtain from (3.6) the heat transport equation for convection:

$$\frac{d\vartheta}{dt} - \frac{\alpha\overline{T}}{C_p\overline{\rho}}\left[\frac{dp}{dt} + \left(\frac{\vartheta}{\overline{T}} - \frac{\rho}{\overline{\rho}}\right)\frac{d\overline{P}}{dt}\right] = \frac{1}{C_p\overline{\rho}}\nabla\cdot k\nabla\overline{T} + \frac{1}{C_p\overline{\rho}}\nabla\cdot k\nabla\vartheta + \frac{Q}{C_p\overline{\rho}} \tag{3.10}$$

This equation is applicable to both the perfect gas for which $\alpha T = 1$ and for liquid for which $\alpha T << 1$. For liquid this equation can be essentially simplified if we take into account that due to (2.11) $\rho/\overline{\rho} \sim \alpha\overline{T}\left(\vartheta/\overline{T}\right) << \vartheta/\overline{T}$ and $\rho/\overline{P} \sim \alpha\overline{T}\left(\vartheta/\overline{T}\right) << \vartheta/\overline{T}$ and neglect the small pressure and density terms. Then having in mind that $d\overline{P}/dt = \mathbf{V}\cdot\nabla\overline{P} = V_r d\overline{P}/dr = V_r d\overline{\Psi}/dr = -V_r\overline{\rho g}$ we obtain:

$$\frac{\partial\vartheta}{\partial t} + \mathbf{V}\nabla\cdot\vartheta + \frac{\alpha\overline{g}}{C_p}\vartheta V_r = \frac{1}{C_p\overline{\rho}}\nabla\cdot k\nabla\vartheta + \frac{Q}{C_p\overline{\rho}} + \frac{\overline{Q}}{C_p\overline{\rho}} \tag{3.11}$$

where

$$\frac{\overline{Q}}{C_p\overline{\rho}} = \frac{1}{\overline{\rho}C_p r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial\overline{T}}{\partial r}\right) \sim \frac{1}{r^2}\frac{k}{C_p\overline{\rho}}\frac{\partial}{\partial r}\left(r^2\frac{D\overline{T}}{d}\right) \sim Dk\frac{\overline{T}}{d^2} \tag{3.12}$$

Let us rewrite (3.11) in a form:

$$C_p\bar{\rho}\frac{\partial\vartheta}{\partial t}+\nabla\cdot\left(\mathbf{V}\vartheta C_p\bar{\rho}\right)+\alpha\overline{g\rho}\vartheta V_r=\nabla\cdot k\nabla\vartheta+Q+\bar{Q}\qquad k=\frac{k}{C_p\bar{\rho}}\tag{3.13}$$

The superadiabatic temperature and the flow velocity can be divided in two parts: the large scale values and the turbulent ones with zero mean values:

$$\mathbf{V}=\bar{\mathbf{V}}+\mathbf{V}_T\qquad \vartheta=\bar{\vartheta}+\vartheta_T\qquad \overline{\vartheta_T}=0\qquad \overline{\mathbf{V}_T}=0$$

Averaging (3.13) over a small space scale we rewrite it in a form:

$$C_p\bar{\rho}\frac{\partial\bar{\vartheta}}{\partial t}+\nabla\cdot\left(\bar{\mathrm{V}}\bar{\vartheta}C_p\bar{\rho}\right)+\alpha\overline{g\rho}\left(\bar{\vartheta}\bar{V}_r+\overline{\vartheta_T V_{rT}}\right)=$$
$$\nabla\cdot\left(k\nabla\bar{\vartheta}-\overline{\mathrm{V}_T\vartheta_T}C_p\bar{\rho}\right)+Q+\bar{Q}\tag{3.14}$$

As usual we adopt that the term $-\overline{\mathrm{V}_T\vartheta_T}C_p\bar{\rho}$ describes the turbulent diffusion and can be written in a form:

$$-\overline{\mathrm{V}_T\vartheta_T}C_p\bar{\rho}=k_T\nabla\bar{\vartheta}$$

where $k_T>>k$. We rewrite the term $\alpha\overline{g\rho}\,\overline{\vartheta_T V_{rT}}$ by analogy with the previous term in a form:

$$\alpha\overline{g\rho}\,\overline{\vartheta_T V_{rT}}=\frac{\alpha\bar{g}}{C_p}\overline{\vartheta_T V_{rT}}\,\bar{\rho}C_p=-D\frac{k_T}{d}\frac{\partial\bar{\vartheta}}{\partial r}$$

We assume that k_T does not depend on the coordinates. Then dividing (3.12) by $C_p\bar{\rho}$ we obtain the heat transport equation in its final form:

$$\frac{\partial\vartheta}{\partial t}+\left(\mathrm{V}\nabla\right)\vartheta+D\frac{\vartheta V_r}{d}=k_T\nabla^2\vartheta+D\frac{k_T}{d}\frac{\partial\vartheta}{\partial r}+\frac{Q}{C_p\bar{\rho}}+\frac{\bar{Q}}{C_p\bar{\rho}}\tag{3.15}$$

For simplicity we omit here the bars over V and ϑ.

This equation is in agreement with our intuitive expectations in the Introduction that β-term is absent. All new comparative to Boussinesq approximation terms through $\bar{Q}$ (see (3.12)) are proportional to D and vanish in incompressible limi $D\rightarrow 0$.Nevertheless this term can be neglected only if the superadiabatic heat flux on the bottom boundary is much larger than the adiabatic heat flux on the upper boundary. This condition is not true for the liquid Earth core.

Boussinesq approximation supposes that the only source of density variations and consequently the buoyancy force are variations of temperature. However the momentum equation (3.3) includes a term proportional to the pressure:

$$\bar{\rho}\nabla\left(p/\bar{\rho}\right)=\nabla p-1_r\left(p/\bar{\rho}\right)d\bar{\rho}/dr$$

Taking (3.8) into account one can see that this term is of order of D and (3.3) can be rewritten in a form:

$$\bar{\rho}\frac{d\mathbf{V}}{dt} = -\nabla p - 1_r \frac{p}{d}\frac{D}{\gamma} + 1_r \overline{g\rho}\alpha\vartheta - 2\bar{\rho}\Omega\times\mathbf{V} + \mathbf{F}^b + \mathbf{F}^\upsilon$$

The deviation of the continuity equation from the equation of incompressibility is also of the order of D:

$$\nabla\cdot\mathbf{V}\bar{\rho} = \bar{\rho}\nabla\cdot\mathbf{V} + V_r\frac{d\bar{\rho}}{dr} = \bar{\rho}\left[\nabla\cdot\mathbf{V} + V_r\frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dr}\right] = \bar{\rho}\left[\nabla\cdot\mathbf{V} - \frac{V_r}{d}\frac{D}{\gamma}\right] = 0$$

Energy balance

Multiplying (3.15) by $C_p\bar{\rho}$ we obtain the equation for internal energy $\varepsilon^\vartheta = C_p\rho\vartheta$ corresponding to the superadiabatic temperature:

$$\frac{\partial\varepsilon^\vartheta}{\partial t} = \nabla\cdot\left[-\mathrm{V}\varepsilon^\vartheta + k_T\nabla\vartheta + 1_r\frac{\partial\bar{T}}{\partial r}\right] - \mathrm{F}^a\cdot\mathrm{V} + Q \tag{4.1}$$

The kinetic energy balance can be obtained by multiplying (3.3) by $\mathbf{V}$:

$$\frac{\partial\varepsilon^k}{\partial t} = -\nabla\cdot\left[\mathrm{V}\left[p+\varepsilon^k\right] + \left(\mathrm{V}\sigma'\right)\right] + \mathrm{F}^a\cdot\mathrm{V} + \mathrm{F}^b\cdot\mathrm{V} - Q_\upsilon \tag{4.2}$$

Finally multiplying (2.6) by $\mathbf{B}/\mu_o$ we obtain the equation for the magnetic energy balance:

$$\frac{\partial\varepsilon^m}{\partial t} = -\nabla\cdot\frac{\mathrm{B}\times\mathrm{E}}{\mu_o} - \mathrm{V}\cdot\mathrm{F}^b - Q_j \tag{4.3}$$

where the viscous and the Ohmic dissipations are defined by the expressions:

$$Q_\upsilon = \sigma'_{ik}\frac{\partial V_i}{\partial x_k} \qquad\qquad Q_i = \frac{\mathrm{J}^2}{\sigma}$$

and σ' is the tensor of viscous tensions.

$$\frac{\partial\left(\varepsilon^\vartheta + \varepsilon^k + \varepsilon^m\right)}{\partial t} =$$

$$-\nabla\cdot\left[\mathrm{V}\left(\varepsilon^\vartheta + \varepsilon^k + p\right) + \left(\mathrm{V}\sigma\right) + \frac{\mathrm{B}\times\mathrm{E}}{\mu_o} + k_T\nabla\vartheta + \mathrm{I}^a\right] + Q - Q_\upsilon - Q_j \tag{4.4}$$

The sum of (4.1), (4.2) and (4.3) equations gives

If additional heat sources are absent then all changes of the energy of a liquid element can be only due to the flux across its walls. This means that the rhs of (4.4) has only to be a divergence. So $Q=Q_v+Q_j$.
The integral form of the balance equations can be obtained integrating (4.6) over space. Integrating over volume of the outer core we obtain

$$\frac{\partial E}{\partial t} = 4\pi r_2^2 \langle I(r_2)\rangle - 4\pi r_1^2 \langle I(r_1)\rangle, \tag{4.5}$$

where the brackets $\langle \ldots \rangle$ denote averaging over angles θ, ϕ :

$$\langle A \rangle = \frac{1}{4\pi}\iint A \sin\theta d\theta d\phi.$$

and E is the whole energy of the core:

$$E = E^{\vartheta} + E^{k} + E^{m} \tag{4.6}$$

Here $E^{sa} = E^{m} + E^{k}$ are the whole internal, magnetic and kinetic energies of the liquid core:

$$E^{\vartheta} = C_p \iiint \bar{\rho}\vartheta d^3 r, \qquad E^{m} = \iiint \frac{\mathbf{B}^2}{2\mu_o} d^3 r, \qquad E^{k} = \iiint \frac{\bar{\rho}\mathbf{V}^2}{2} d^3 r, \tag{4.7}$$

and $I(r)$ is the radial component of the surface density of the energy flux across the boundary. The whole energy flux contains a lot of terms but the main part of them vanishes on the boundaries due to the no-slip boundary condition $V(r_{1,2})=0$. We can additionally simplify the situation if we expand the integrating volume of the magnetic energy to the whole space. Since $\mathrm{B}\times\mathrm{E}/\mu_o$ decreases faster than r^{-2} on the infinity, the flux of electromagnetic energy vanishes and the energy flux taking part in the energy balance is reduced to adiabatic and superadiabatic heat flux only:

$$I(r) = I^{sa}(r) + I^{a}(r) \qquad I^{sa} = -k_T \frac{\partial \vartheta}{\partial r}, \qquad I^{a} = -k \frac{\partial \bar{T}}{\partial r} \tag{4.8}$$

Eq. (4.5) tells us that the whole energy changes only due to the difference between the two heat fluxes on the boundaries. The heat flux enters across the bottom boundary and leaves the volume through the upper boundary. This picture is very similar to that described by (1.6) of the Boussinesq approximation. However there is a crucial difference between (1.6) and (4.5) in the lhs of these equations. In (1.6) this is only internal energy, but in (4.5) this is the whole energy including internal, kinetic and magnetic energies. This could be expected since Arhimedean work supporting the kinetic and the magnetic energies is of order of D and speaking strictly it vanishes in the incompressible limit $D \to 0$.

To make our statement clearer we slightly simplified it neglecting the movement of the boundaries. Having in mind the appliance to the Earth core we also have to take into account the rotation of the mantle and the inner core. Then the kinetic energies of both bodies have to be added to the whole energy. And also the viscous energy flux connected with the non-zero velocity on the boundaries must be taken into account.

(Velocity would be $\mathrm{V} = \omega \times \mathrm{r}$, where r is a radius vector on the point situated on the boundary and is its angle velocity).

Another difficulty is of a more principal character. In computer simulations the momentum equation implies to be averaged over turbulent scales since viscous scales are too small to be resolved. It means that the Arhimedean work in the momentum equation is created only by the averaged superadiabatic temperature and has a form: $\alpha \overline{g\rho\vartheta V_r}$. However the same work in the equation for the internal energy comprises one extra term $\alpha g\rho \mathrm{V}_T \vartheta_T$. That term is not canceled when summing up the equations of partial energies (4.1-3) to obtain the equation for the whole energy.

This means that a sub-system of turbulent eddies has to be taken into account. Its energy balance equation takes a form:

$$\frac{\partial \varepsilon_T^k}{\partial t} = -\nabla \cdot \left[\overline{\mathrm{V}_T p_T + \mathrm{V}\varepsilon_T^k} + \overline{(\mathrm{V}_T \sigma_T')} \right] + \alpha \overline{g\rho} \overline{\mathrm{V}_T \vartheta_T} - \overline{\sigma_{ikT}' \frac{\partial V_{iT}}{\partial x_k}}$$

This equation has been obtained in assumption that the magnetic scales are resolved by computer simulation and respectively the magnetic terms must not be included in the turbulent energy.

This equation must be added to (4.1-3) to eliminate the problem with the additional Arhimedean work. Additional turbulent dissipation has to be included in Q.

Discussion

Let us rewrite the convection equations in a slightly changed form

$$\overline{\rho} \frac{d\mathrm{V}}{dt} = -\nabla p - 1_r \frac{p}{d}\frac{D}{\gamma} + 1_r \overline{g\rho}\alpha\vartheta - 2\overline{\rho}\Omega \times \mathrm{V} + \mathrm{F}^b + \mathrm{F}^\upsilon$$

$$\nabla \cdot \mathrm{V} = \frac{V_r}{d}\frac{D}{\gamma} \qquad (5.1,2)$$

$$\frac{\partial \vartheta}{\partial t} + (\mathrm{V}\nabla)\vartheta + \frac{\alpha \overline{g}}{C_p} V_r \vartheta =$$

$$k_T \nabla^2 \vartheta + D\frac{k_T}{d}\frac{\partial \vartheta}{\partial r} + \frac{Q}{C_p \overline{\overline{\rho}}} + \frac{\overline{Q}}{C_p \overline{\overline{\rho}}} + \frac{\overline{Q}_j}{C_p \overline{\overline{\rho}}} + \frac{\overline{Q}_\upsilon}{C_p \overline{\overline{\rho}}} \qquad (5.3)$$

and discuss the results obtained.

Boussinesq approximation supposes that the only source of the density variations and consequently the buoyancy force are variations of temperature. In our Adiabatic approximation the momentum equation (5.1) includes a term proportional to pressure. If $p<0$ i.e., the whole pressure is smaller than the equilibrium one, then density increases and the correspondent buoyancy force is positive. It is negative in the opposite case. Eq. (5.2) shows that an ascending liquid element increases its volume and decreases it when traveling down. Both these effects vanish in incompressible limit $\aleph \to 0$.

Convection exists only when the temperature of the Reference state is the adiabatic and not an arbitrary one as it is sometimes assumed in some papers. This adiabatic temperature is defined by the equation (3.7) which generally speaking, does not satisfy the condition of the constant adiabatic heat flux $\nabla \cdot k\nabla \overline{T} = 0$. That is why in order to be stationary RS needs an energetic support with space density equal to $C_p \overline{\rho} \nabla \cdot k\nabla \overline{T}$. This heat is taken from the superadiabatic heat flux. That is why it plays the role of cooling in (5.3) (the fourth term in rhs). This cooling is not negligible since it eats up about 3/4 of the superadiabatic heat flux.

As this has already been said in the Introduction, a rising liquid element changes its adiabatic temperature to the adiabatic temperature of its new environment. That is the reason why the large β-term which in some papers is taken as equal to $V_r d\overline{T}/dr$ is absent in (5.3).

The driving force of the convection is the buoyancy one. It compensates the viscous and the Ohmic dissipation. It takes energy for this from the superadiabatic heat flux cooling it. The term describing this cooling which can be called Arhimedean cooling is the third term in the lhs of (5.3). To understand deeper its role let us integrate the kinetic energy equation over space and average the equation obtained over time. As a result we obtain that the whole Ohmic and viscous dissipations are compensated by the whole work of the Arhimedean force. The same role in this equation is played by the third term in lhs (5.3). Without this term dissipations would convert into additional heating source in the heat transport equation in contradiction to the energy law conservation. That is why we cannot use the heating due to these dissipations in Boussinesq approximation and on the contrary we cannot neglect them in the present approach. Since the locations of the Ohmic heating, generally speaking, do not coincide with the locations of the Arhimedean cooling we neglect some interesting dynamics using the Boussinesq approximation.

The last cooling term is the second one in the rhs of (5.3). The momentum averaged over turbulent scales describes the viscous dissipations connected with the turbulent viscosity. This dissipation leads, as we discussed above, to the Arhimedean cooling. However it does not describe the dissipations on the non-resolved turbulent scales and the correspondent cooling in the heat transport equation. The second term in (5.3) describes just this cooling which can be called the turbulent cooling.

This cooling is not suitable for numerical simulations since it disturbs the energy balance for the averaged over turbulent scales values. We have discussed this in the previous chapter. We think that a rather intuitive reason exists which allows us to neglect this term. Let us discuss it starting from another topic.

Magnetic field is supplied by energy at one location but it can spend this energy by the Ohmic dissipations in other location which can be at a long distance from the first one.

That is why Ohmic dissipations do not compensate locally the energy producing the field in the equation (5.3). They compensate themselves only integrally in space and time. Turbulence is local in its nature. That is why it can be assumed that turbulent eddies spend their energies at the same place where they get them from. Of course there is a time difference between getting and spending of the energy but this time interval is rather small in comparison with the typical times of the convection. Then under averaging over turbulent time scale both the turbulent cooling and turbulent heating are cancelled in (5.3). This assumption is equivalent to averaging of (4.9) over turbulent space and time scales after which this equation is reduced to the balance between the Arhimedean work and the viscous dissipation:

$$\alpha \overline{g\rho}\,\overline{V_{rT}\vartheta_T} = \overline{\sigma'_{ikT}\frac{\partial V_{iT}}{\partial x_k}} \tag{5.4}$$

Thus the convection equations do not depend on the turbulence (except the turbulent transport coefficients, k_T and ν_T). Note that this term cannot arise at all in the thermodynamics approach of Bragynsky and Roberts (1995) (see also Glatzmaier and Roberts (1995)) since entropy does not diffuse and these authors use the turbulent diffusivity.

The commonly used Boussinesq approximation, without Adiabatic cooling and correspondently Adiabatic heat flux (see e.g. Glatzmaier and Roberts (1995)) is in fact the Boussinesq approximation with uniform "Adiabatic temperature". It follows from (3.9) ($D \sim \Delta\overline{T}/\overline{T}$) that for this incompressible case $D \to 0$ and consequently Arhimedean force vanishes from the momentum equation (5.1). Together with the Arhimedean force Arhimedean work and respectively the kinetic and the magnetic energies are of order of $O(D)$comparatively to the internal energy. So they are absent in the Boussinesq energy equation (1.6).

How can an existing Boussinesq code be fitted to the presented equations? The main term which has to be taken into account is the Adiabatic cooling in the heat transport equation. Of course the β-term must be removed if it exists in the code. These are absolutely required changes but they are not complicated ones.

It may seem as a paradox but the direct effects of compressibility, the pressure term in (5.1) in the form $\overline{\rho}\nabla\left(p/\overline{\rho}\right)$ and the continuity equations $\nabla \cdot \mathrm{V}\overline{\rho} = 0$, are not so obligatory. We can say that $D\sim 1/4$ is small enough to neglect their differences with the incompressible values ∇p and $\nabla \cdot \mathrm{V} = 0$.

Arhimedean cooling is the term of order $O(\aleph)$ in the heat transport equation. Nevertheless it is desirable for it to be taken into account in the program due to two reasons. The first of them is that together with this term we must consider the dissipations (mainly Ohmic) which can introduce an interesting dynamics in the convection especially in the currents layers.

The other reason is connected with the possibility to check the work of the code. Even the Boussinesq energy balance gives this possibility. We can evaluate the lhs and rhs of (1.6) and compare them. Their coincidence confirms that the solution is correct. However in the Boussinesq's case we check the solution only of the heat transport problem which is almost trivial since it includes the only equation. If we test the whole energy balance in form

(4.5) we check the solution of the whole problem with all equations and boundary conditions.

To realize this idea let us introduce a new function Υ, the Imbalance:

$$\Upsilon = \frac{\partial E}{\partial t} - 4\pi r_2^2 \langle I(r_2) \rangle - 4\pi r_1^2 \langle I(r_1) \rangle, \tag{5.5}$$

According to (4.5) Υ has to vanish. This equality however is not trivial and is only satisfied by the true solution of the problem. That's why the Imbalance, Y, can be used as a check of the solution.

In computer simulations the equality $\Upsilon = 0$ cannot be achieved. Even for correct codes there are errors due to truncations, relatively large space and time steps, and so on. So always has a non-zero value. If however this value is small (comparative to the main term included in Υ) the solution is good.

It is natural to normalize the Imbalance by the Adiabatic heat flux $4\pi r_2^2 I^a(r_2) \sim 5\times 10^{12} W$ which is a typical large value included in the energy flux I. To use the Imbalance to test the numerical solution we must evaluate numerically the time derivative from the total energy E and values of energy fluxes on both boundaries. Then we evaluate the Imbalance and normalize it by $I^a(r_2)$. If its value is small (say smaller than 0.01) then the solution is good. (It is important to emphasize that just including the terms of the first order in D expansion makes the Imbalance interesting for the problem. In the main approximation it is reduced to almost trivial heat flux balance.)

SEDI's benchmark proposes the method of testing results based on their comparison at the end of the fixed time interval, whereas Imbalance can be applied continuously at every instant. By comparison we cannot estimate the truncation and time and space steps errors - they will be the same in different codes. Imbalance estimates the solution of the problem including all possible errors. The Imbalance also allows us to gradually complicate the problem by including new terms in the equations and in the energy balance. Finally by evaluating the Imbalance we test our own solution independent of others. That is why we assume that the Imbalance calculation (few surface and volume integrals) is not a very high price to pay for knowledge about our solution. It was used by Anufriev, Cupal and Hejda (1993) in their code for $\alpha\omega$-dynamos and it helped them to exclude many errors. It was interesting to note enhancement of the Imbalance, when a thin current layer arises in the solution.

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(Received 20.02.2003; accepted 2006)

Адиабатичното приближение при геодинамоконвекцията

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Резюме. Магнитното поле на Земята се създава от конвективни течения в нейното електропроводящо течно ядро. Съответно уравненията за термичната конвекция са съществената част от проблема за генерацията на това поле. Обикновено конвекцията в течното ядро се изучава в приближението на Буссинеск (БП), пренебрегващо свиваемостта на течността. Последната, обаче, играе съществена роля в конвекцията в земното ядро и по-специално в енергетичния му баланс. Без отчитане на свиваемостта този баланс се свежда само до баланса на топлината. Ето затова обобщаването на БП към случая на свиваемата конвекция, изглежда актуално. В настоящата работа, ние представяме уравненията на конвекция за случая на свиваемата течност. В сравнение с Бусинесковото приближение тези уравнения показват няколко характерни особености. В уравнението за топлинен пренос възникват два нови члена, наречени от нас адиабатичното и Архимедовото охлаждания. Първият от тях описва топлина, поддържаща адибатичен профил на температурното разпределение на адиабатичното референтно състояние. Вторият член е свързан с тази част от топлината, която се превръща в механична работа, създаваща магнитно поле. В хидродинамичното уравнение също възниква нов член. Този член описва допълнителната Архимедова сила директно свързана със свиваемостта на течността. Всичките тези нови членове зависят от разликата на плътността между горната и долната граници на ядрото и изчезват, когато тази разлика клони към нула.